MEASUREMENTS OF MULTI-STAGE CHANGE OVER TIME IN SAFETY-CAMPAIGNS

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An effect of safety campaigns may exist if several crucial assumptions are satisfied. In order to trace these assumptions from the end to the starting point of the campaign three assumptions and some deduced requirements are listed.

1.0. Assumptions and Requirements

Assumption 1: Safety is enhanced by a change in behaviour.

The validity of this assumption requires a firm basis for the relation between behaviour and traffic safety. This basis may be given from prior experimentation or observation. Problems arise if one tries to study the relation of behaviour and safety by an introduction of a safety campaign. These problems are located in the confounding intervening variables of the introduction of the campaign itself on the one hand, and on the other hand in the amount and grouping of accident data that is needed for a significant gain in safety. (OECD Reports on road research programmes S4 and S7). So, apriori, it seems a requirement that only such safety campaigns are demonstrable effective which changes behaviour that is already known to contribute to more safety. Still then one must be able to detect a change in behaviour. This asks at least for two experimentally independent and reliable assessments of the actual behaviour. If this is not possible indirect measurement of the behaviour, if generalizable to the actual behaviour, may be used. Without direct or generalizable indirect measurements of behaviour a demonstrable effect of safety campaigns is impossible. Moreover a positive change in behaviour must be possible. In the case where this is questionable because the faulty behaviour is thought to occur by error which are not under control of the driver, the effect of a campaign will not be observable. To summerize we formulate 3 requirements for this assumption to be true.

Requirement 1.1. Relation between behaviour and safety is known

1.2. Independent and reliable assessments of actual behaviour or valid indirect indicators of actual behaviour on at least two occasions.

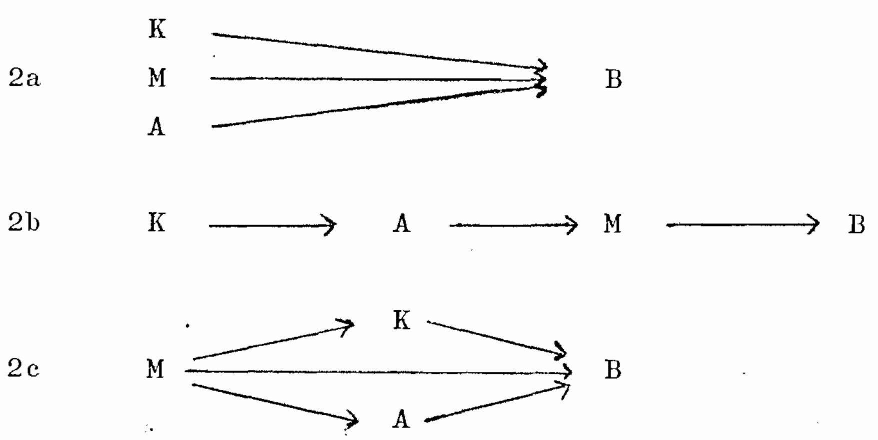
1.3. Existence of an imperfect behaviour that likely can be changed in positive direction by a campaign.

<u>Assumption 2:</u> The change in behaviour is caused by the interrelated changes in knowledge, motivations and attitudes.

The impact of a campaign will be related to aspects of knowledge motivations and attitudes. In order to know what kind of campaign is most effective and how the campaign must be structured with respect to time, content, media and audience, it will be relevant to elaborate this assumption by a structuring of the relations between knowledge, motivations and attitudes. Some of the possible assumptions are formulated in 2a, 2b en 2c.

2a: Changes in knowledge, motivations and attitudes are independent and each causes a part of the change in behaviour.
2b: A change in knowledge causes a change in attitudes which in turn causes a change in motivations and this finally is the cause of change in behaviour.
2c: A change in motivations makes the changes in knowledge and attitudes an effective cause of the behaviour-change but also causes a direct change in behaviour.

One could diagram these assumptions as follows.



Many other diagrams may be possible, the general idea however will be clear. Techniques for the identification and computations of such diagrams will be discussed later on, as path-analysis of correlation matrices. The importance for the content and timing of content of the campaign programmes of such an analysis is evident. The situation is even more interresting if different target-audience groups show different diagrams. A systematic variation of content and presentations for different groups may result in a design from which an optimal campaign can be resolved. But also there, where a fixed campaign programme is presented, it will be very informative to find out the correct causal path. It may give us answers to the questions on how the effects (or ineffective results) of a campaign are obtained.

Path-analysis of correlations may be interpreted as causes if a theory explains such a causal relation. It is legitimate to do so if the time-relation of cause and effect is known and no other rival explanations are possible (Wold, 1956). In the relations between knowledge, attitudes and motivations no time order is known and no clear theory is available. If the correlations are correlations between changes in time for the same observation units a causal interpretation of these correlations is valid, under the condition that the structure of the correlations reveals a particular path. Such causal inferences are a modern realization of the logically based method of concomitant variations for induction (J.S. Mill, 1868).

From assumption and what is discussed here, it is clear that a neglectable effect of campaign may be expected if knowledge, motivations and attitudes are already on a relative high level Target-group selection therefore may be based on this, as also the content of the campaign programme may be based on those aspects of knowledge, motivation and attitudes that are strongly correlated with the wanted behaviour. The measurements needed for such selections and correlation study may be the first measurement of a timeserie in the before campaign period.

The requirements following from assumption 2 and this discussion are:

Requirement 2.1. Independent and reliable assessment of knowledge motivations and attitudes for the same sample at at least two occasions.

2.2. Existence of a shortage of knowledge, motivations and attitudes which are likely to be changed in an adequate state.

Assumption 3: Changes in knowledge, motivations, attitudes and behaviour are caused by the safety campaign.

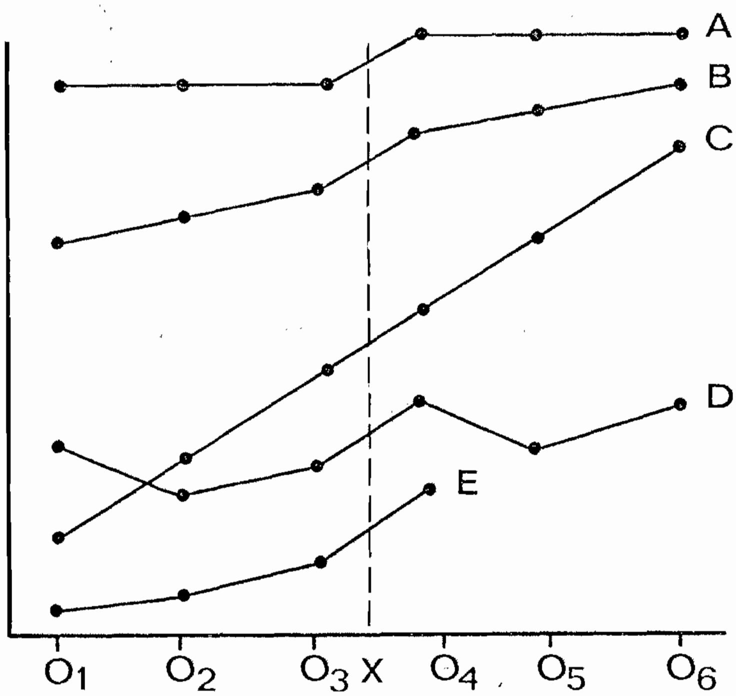
Although an effective campaign only need to change the behaviour in the wanted direction, it seems likely that, if the behaviour is changed only by the campaign, also knowledge, attitudes and motivations must change. If there is no change in knowledge, attitudes and motivations it is at least questionable how the behaviour can be changed by the campaign. In any way, it will be necessary to look for changes in all the four domains. The real problem in assumption 3 is the legitimacy of the causation. Such a causal relation is only legitimate if other reasonable explanations of the changes in knowledge, motivation, attitudes and behaviour are ruled out. Procedures where this is done by the introduction of a control group are discussed by Haskins (1970) and the OECD. report on safety campaigns (Report RR/S4). Many campaigns, however, are programmed, without the possibility of an equivalent control group, like in nation-wide T.V. campaigns.
The only design that is possible in this context and that rules out a great number of rival explanations is given by the interrupted time-serie analysis. This type of research is discussed in the next section, where the usual design of interrupted time-series (Campbell and Stanley, 1963) is adapted and complicated in order to enhance its validity and optimality in this context. The basic idea of such a time-serie analysis is that the control is formed by multiple measurements of equivalent or identical samples in the before and after periods, where the changes are not influenced by the "experimental" condition. So the requirements following from assumption 3 are:

Requirement 3.1. Knowledge, attitudes, motivations and behaviour must be measured before and after the time interval of the campaign.

3.2. Either measurements on another uneffected equivalent group at the same occasions or multiple measurements on identical or equivalent groups in both before and after periods.

2.0. Methodology and design of interrupted timeseries

Assuming that only one variable represents the effect of the campaign, then the effect is deduced from the means of this variable in the timeserie.
Several possible outcomes are reflected in the graph below (adapted from Campbell (1963).



Possible outcome pätterns of a timeserie $0_1 - 0_6$ with the experimental variable introduced at x.

It is evident that any effect due to the experimental variable (the campaign) is unjustified in cases E, D and C although the change at x is equal to that of A and B.
Compared with the simple before and after study (0_3 x 0_4) we

control in a timeserie for trend changes (C, E) fluctuations over time (D) and the main effect of repeated measurement. We assume the measurements to be taken from a random sample or eventually a serie of equivalent random samples, therefore no problems of selective sampling or differential recruitment need to be considered. There are, however, two main problems, which are not ruled out by a simple time serie.

Firstly one must be sure that no other condition or variable that is relevant for the measurement of the effect variable is changed at the interval of the campaign. Such changes form other rival explanations of the effect. If the effect variable is the number of accidents (which is not recommended, see assumption 1) such rival explanations usual will occur as weather conditions or traffic regulations. If the actual behaviour or knowledge is the relevant variable such unwanted simultaneous changes are perhaps less inevitable. Since safety campaigns often accompany a change in law, one must be sure that the effect of the change in law and the campaign is separable, either by control group design or by seperating the change in law and the campaign in terms of the interval in time in which they take place. In the case of a timeserie design any second possible confounding cause is a real threat to the validity of the conclusions and therefore must be eliminated at forehand or seperated from the campaign interval, if possible, by at least one "natural" interval

The second, but repairable weaknes of the time-serie design in this context is the reactive interaction of the repeated measurements on the measurement of the effect of the campaign. If the measurements are not unobtrusive, which is likely the case for the assessment of knowledge, motivations and attitudes and perhaps for the behaviour too, it may be hypothesized that the foregoing measurements make the sampled persons more responsive to the campaign. A somewhat complicated design may obviate this by introducing a timeserie that is partially or as a whole based on equivalent samples not measured before. In the diagrammed representations below three relevant designs are given for a timeserie. Other designs are also possible, but these seem to have some optimal properties.

Random samples	Time 1	2	3	Axis x	4	5	6	Design
1	0_{11}	0_{12}	0_{13}		0_{14}	0_{15}	0_{16}	One sample - repeated measurement
1	0_{11}	-	-		-	-	-	Seperate samples - succesive measurements
2	-	0_{22}	-		-	-	-	
3	-	-	0_{33}		-	-	-	
4	-	-	-		0_{44}	-	-	
5	-	-	-		-	0_{55}	-	
6	-	-	-		-	-	0_{66}	
1	0_{11}	0_{12}	-		-	-	-	seperate samples - succesive overlapping pairwise repeated measurements
2	-	0_{22}	0_{23}		-	-	-	
3	-	-	0_{33}		0_{34}	-	-	
4	-	-	-		0_{44}	0_{45}	-	
5	-	-	-		-	0_{55}	0_{56}	

As far as only means of one variable is concerned, it will be possible to replace the outcome of the repeated measurements by the seperate sample - succesive measurements, only introducing the sampling error which is a function of the number of the sampled observations. In the case where measurements of many variables are combined in change factors or if we want to analyze the relations between changes in knowledge, motivations attitudes and motivations, as was suggested, we need correlations between changescores. This implies at least changescores from one measurement to a next measurement, so the seperate sample - succesive overlapping pairwise repeated measurement design is proposed. This design anables one to seperate the reactive interaction effect and the main campaign effect by comparing the rowwise uneffected mean differences (0_{11}-0_{12}; 0_{22}-0_{23}; 0_{44}-0_{45} and 0_{55}-0_{56}) and the experimentally effected mean rowwise difference ($0_{33}0_{34}$)

with the columnwise uneffected differences ($0_{12} - 0_{22}$; $0_{23} - 0_{33}$; $0_{45} - 0_{55}$) and the possible reactive interation effect in the columnwise difference ($0_{34} - 0_{44}$). Such a comparison can also be formulated as a timeserie of first measurements tested for a difference with the, one time interval lagged, second measurements timeserie, both viewed as a seperate sample succesive measurement design.

3.0. Multivariate Analysis

So far we assumed only one variable for the measurement of an effect of the campaign. Usually, however, the domain of knowledge, motivations and attitudes and probably the behaviour too, are measured by sets of many variables. For researchers in the field of safety research this will form the main problem in the analysis of the data. Although at first glance the use of sets of variables may give troubles, in fact it is a great advantage in a field, where individual variables are unreliable and the relevant main factors in the field are inknown. A multivariate analysis of the data may combine the individual unreliable variables into relevant main factors which are much more reliable. Like the IQ is a factoranalytic result of the analysis of several unreliable items. In our case we are looking for change factors that combines the convergences of the differences in the campaign interval in contrast to the before and after intervals. In order to describe such an analysis, we have to elaborate somewhat on the aspect that may be regarded as a change and what will be viewed as convergent information of such changes.
Every set of variables is statistically, under the restriction of a multinormal distribution and their linear relationships, fully described by the means of the variables, the variances of the variables and the correlations between the variables. A change of the measurements of one occasion to another occasion of sets of variables may occur for the means, the variances, the intercorrelations within a set and the correlations between sets. In terms of changescores this would mean, non-zero mean change for the variables, variances of changescores larger

than the expected error variance, correlations between change scores that are different from the first measurement correlations between the variables and different from correlations of other sets of changescores. In the proposed design this would be translated as:

a) mean changescores in the campaign interval differ from the mean changescores in the before and after intervals

b) the variances of the changescores in the campaign interval are larger than the variances of the changescores in the before and after intervals

c) the correlations of the changescores in the campaign interval differ from the correlations of the changescores in the before and after period

d) the correlations of the changescores in the campaign interval differ from the pooled correlations of the variables in the before period measured at each occasion.

Convergent information of these changes that might be regarded as cumulative evidence is obtained if comparable mean changes take place in variables with a similar meaning. Operationally this means comparable changes for correlated variables. Such groups of correlated variables which comparable mean-changes may be regarded as a result of a change in a latent common change_factor. Since the change may be present in factors that were not present in the individual differences before the campaign (see: d) , it is necessary to take the correlations between the changescores as evidence for the similarity in meaning with respect to the change and not the correlations of the original pre-campaign measurements. If variances form a source of cumulative evidence, then surely the variables with comparable mean change and relative high correlations between changescores, will show also larger changescore variances.

Several multivariate techniques are available for optimization of the aspects of change under a, b, c and d. These techniques are discriminatory analysis (for a), principal component analysis and canonical correlation analysis (for b, c and d). (Anderson

1958, Morrison 1968). These multivariate solutions consist of optimal combination of variables by weighted summation of variables. Since seperate optimization of these change aspects will give different weighting procedures, no convergence of information is obtained. We therefore ask for one weighting procedure that optimizes the different aspects of change together. This might be formulated as a multivariate analysis of differences and covariances of the original multiple sets of measurements (Horst, 1963), but turns out to be rather complicated and yet unsolved. Another approach is the multivariate analysis of the changescores as an adaption and modification of the so-called incremental R-technique (Cattell, 1963).

Since changescores do have a meaningful zero scale point we may combine aspects a, b and c in one statement:
- the crossproducts of the changescores in the campaign interval differ from the crossproducts of the changescores in the before and after period intervals -
This statement combines a, b and c, because tbe crossproducts will be larger for variables with comparable mean-change, with larger variances and with relative higher correlations. Aspect d) need not to be maximized because there is nothing against the identity of factors for change and individual differences at one occasion. The above statement naturally leads to the multivariate technique outlined in Appendix A. It asks for the maximum of the ratio of two symmetric quadratic forms and is because of the similarity with the canonical analysis of discriminance (Porebski, 1966) called canonical analysis of increase.
A popular frasing of the end-result of the analysis could be that the analysis combines the changescores into latent common-changefactors, which, in the timeserie, show maximal discontinuity and change in the campaign interval and minimal discontinuity and change in the other intervals. The test of significance is identical with the well known Λ test for canonical analysis, so an overall significance test is also available (Bartlett1947).

The canonical analysis of increase may be applied seperately to the sets of knowledge variables, motivation variables, attitude

variables and behaviour variables, and that is what is proposed here. Because of the search for causal relations we could have asked for such a weighting procedure that the correlations between these sets of changescores were maximized. Such a weighting, however, might have the disadvantage, that not all the information of the change is analysed because of a lack of correlation with another set of variables. Therefore, it seems justified, first to analyze the significant change factors caused by the campaign and to correlate the changefactors of different sets afterwards. If the analysis yields more than one significant dimension of change in a set of variables, we may rotate these dimension to a simple structure, like it is done in factoranalysis (Harman, 1960). Such rotated factors are interpretable as meaningful (psychological) factors of change. The causal relation between these meaningful factors are to be shown, not to be maximized.
The same thing applies to the relations between the changefactors and the individual difference factors in the one occasion measurements of the before period: it is up to the empiric correlations between them to establish what the similarities are.

If it is hypothesized that the changefactors are likely similar to the individual difference factors and it is assumed that the relations between these individual difference factors of different sets are also similar, then one may use these between set relations of the first measurement of the serie for the construction of the most effective campaign content. A canonical correlation analysis (Anderson 1958, Carroll 1968) gives the components in the behaviour which are maximally correlated with the components in the domain of knowledge, motivation and attitudes. These canonical correlation components of knowledge, motivation and attitudes variables are to be represented in the content of the campaignmessages, if this similarity between changefactors and their between set relations and individual difference factors and their between set relations is true

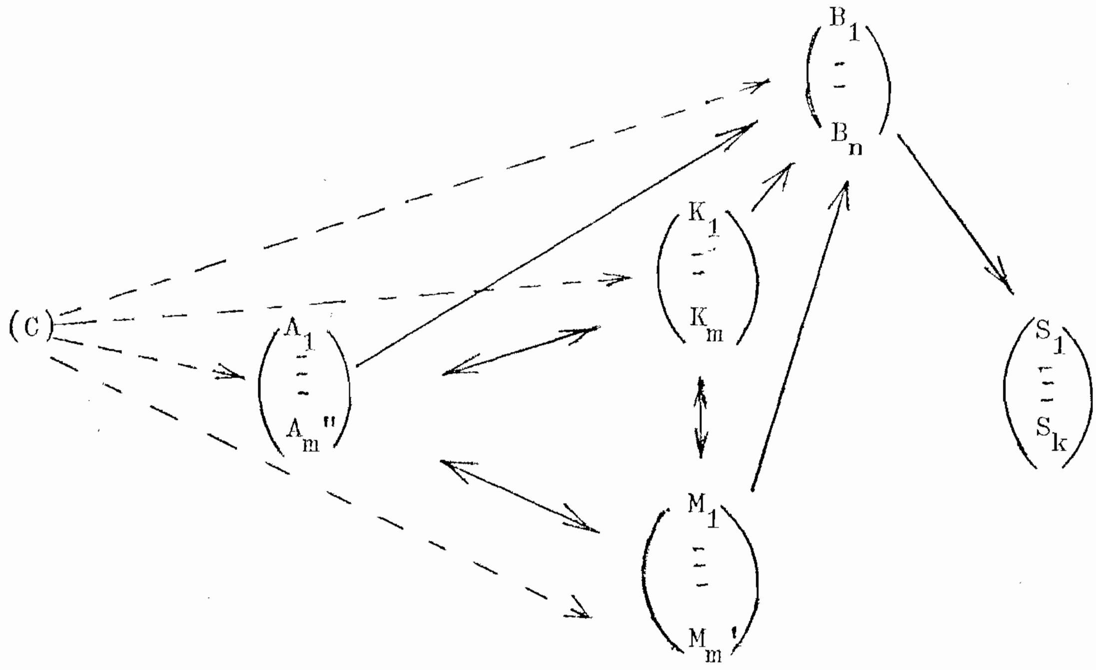
A last question on the multivariate analysis of the data has to be raised. In the case we find no significant change in knowledge,

motivations and attitudes, but a significant change in behaviour, would this mean that knowledge, motivations and attitudes were not relevant for a behavioral change? The answer will depend on the possibility of relations with attitudes, motivations and knowledge variables, which were not observable from the changescores. Such a possibility may exist if the behavioural change by the campaign, brings the behaviour more in consistency with the unchanged, already existing knowledge, motivation and attitude structure. Specially if knowledge and attitudes are already on a desired level, but dissociated from the behaviour, this may occur. A behavioural change that is caused by a consonantal association with knowledge- and attitude structures can be demonstrated by the comparison of the canonical correlations of the behaviour variables and the knowledge, motivation and attitude variables in the before period measurements and the after period measurements. Since these changes in consonance may also occur if there are changes in knowledge, motivation and attitudes it is desired to perform these canonical correlation analysis anyway.
It also will be very informative if the changefactors of the canonical analysis of increase are decomposed in its different sources, stated under aspects a, b and c, regarding the means, variances and correlations.

4. Identification and quantification of causal relations

The general idea of the causal structure may be pictured by directed arrows as shown below.
(C) denotes the campaign; $(K_1 \ldots K_m)$; $(M_1 \ldots M_{m'})$; $(A_1 \ldots A_{m''})$ and $(B_1 \ldots B_n)$ represent the possible significant changefactors for the respective domains of knowledge, motivations, attitudes and behaviour. For the sake of completion the possible change aspects of safety are denoted by $(S_1 \ldots S_k)$. The unidirected broken arrows indicate the causal effects of the campaign, whose existence is to be demonstrated by the canonical analysis of increase and further analysis. The unidirected solid arrows are the possible causal relations from which the time-order of cause and effect is assumed to be known, while the bidirected solid arrows are the

twc possible causal relations from which at least one direction has to be dropped on the basis of further analysis or it may indicate a recursive dependence relation without a causal interpretation.



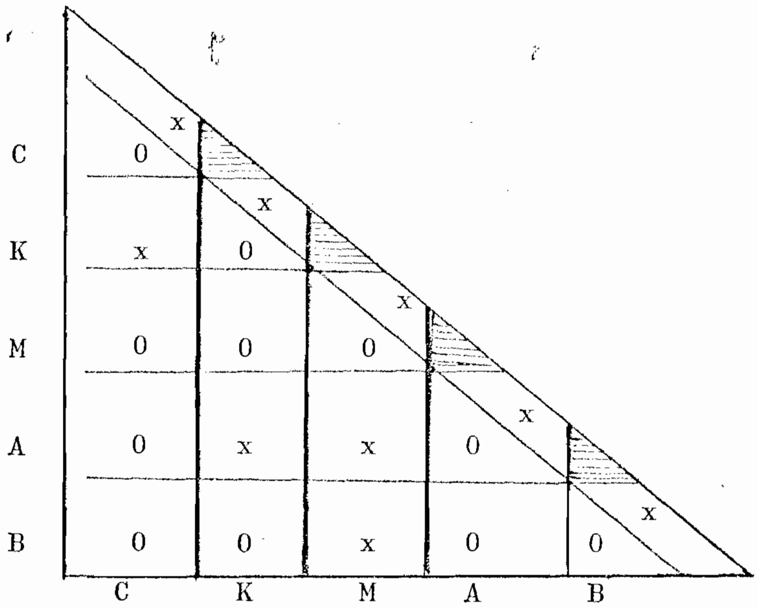
General graph of possible causal relations

The general graph includes the three assumptions of section 1. (assumption 2a, 2b and 2c), which of these or other possible causal structures is involved, is a matter of further analysis of the relationships between the campaign and the interrelated significant changefactors of the different domains. In this causal structure analysis, we have to identify, and to test in a quantitive way, which relations may be explained as a result of two or more other relations. For example can the relation (C) - (B) for a behaviour changefactor totally be explained by the relation of the campaign with the changefactor of knowledge and knowledge-with behaviourfactors? If this is the case and the reverse is not, then we will take the causal relation to follow the path (C)→(K)→(B) and no direct influence on behaviour. Which causal structure is accepted will depend on the identification of the most parsimonious path that is sufficient to explain the other relations and that is meaningful in terms of assumed time order and possible theoretical explanation.

In order to come to an identification and quantitive demonstration of the causal path, we have to construct the matrix of the relations between the campaign, knowledge, motivation, attitude and behaviour (and eventually the safety) factors. Since we are concerned with relations of changefactors based on raw change-scores with a meaningful zero-scale point as the origin, while the variance of these changefactors is independent of the causal inferences, we construct a matrix of congruence coefficients as the normalized crossproducts of the factorscores for the changefactors of the different sets. The congruence coefficients for the different changefactors and the campaign variable are computed from the "variance" ratio's of the canonical analysis of increase. If, in the case of multiple changefactors within a set, the rotation of the factors to a meaningful simple structure is orthogonal, then the congruence coefficients between factors of each set will be zero and no relational analysis between them will complicate the analysis. The mathematical definitions of these congruence coefficients are given in appendix A, the meaning of them is quite the same as for correlation coefficients, except that it does not represent the relations between deviations from the mean but from the zero-change point. The general ideas of structural analysis as path analysis (Blalock, 1961; Boudon, 1967), radex-analysis (Guttman, 1955) and other related linear algebraic analysis techniques (van de Geer, 1970), therefore equally apply to the congruence matrix of the changefactors.

The identification of the causal structure is most effective determined from the inverse of the congruence matrix. The off-diagonal elements of this inverse, normalized by the diagonal elements of this inverse, are the partial congruence coefficient after all the remaining (n-2) factors are held constant. So if some of these off-diagonal elements are close to zero and the corresponding diagonal elements are relative high, then the relations between these factors can be explained fully by the relations of the other factors.

If for example the only real departures of zero inverse elements indicated by an x are located as shown in the figure below, then the so-called simplex structure (Guttman, 1955) is present and the hypothesis of assumption 2b as the causal order of C ⟶ K ⟶ A ⟶ M ⟶ B (⟶ S) is verified



Simplex structure of inverse matrix corresponding to assumption 2b.

If more elements are markedly non-zero then one has to complicate the identification of the causal structure. In general we may drop, the arrows of the general graph of above, if there are near-zero values in the inverse congruence matrix. The remaining arrows can be quantified by the hypothesized relations as weights for the regression or partial regression of the factors, in which the behaviour changefactor are the terminals of the causal chain.

For the possible computational methods and their relations reference to Van de Geer is made (Van de Geer, 1970). If a clear causal structure of a parsimonious and theoretical expectable nature is detected, the computations for a quantification of the structure, reduces to the determination of the zero relations of omitted arrows and the simultaneous solution of the Y unknown weights, corresponding to the non-cancelled arrows, from a set of linear equations containing the $Y = (n-1)n/2 - x$ known congruence coefficients.

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Appendix

Canonical Analysis of Increase

Let D_{ik} denote the differencescore matrix of sample i (i = 1..e..r) and domain k (k→ K = knowledge, M = motivations, A = attitudes, B = behaviour) from the second measurements minus the first measurements on the N_i individuals and m_k variables, according the separate sample - successive overlapping pairwise measurements design in time-series, where for i = e = (r+1)/2 the experimental condition is changed in the time-serie.

We ask for a matrix of weights W_k for each domain such that the quadratic norms of the weighted sums of the differencescores are maximized for i = e and minimized for i ≠ e, since in that case the discontinuity and change is maximized in the experimental time interval e with respect to the other time intervals of the before and after period. So we require

$$W'_k \; D'_{ik} \; D_{ik} \; W_k = \max \quad ; \; i = e$$

$$W'_k \; D'_{ik} \; D_{ik} \; W_k = \min \quad ; \; i = 1....r \; ; \; i \neq e$$

under the condition $W'_k \; W_k = I$

$$\text{If } \operatorname{tr} \left(W'_k \left[\sum_{\substack{i=1 \\ i \neq e}}^{r} \frac{N_i}{\sum N_i} D'_{ki} \; D_{ki} \right] W_k \right) = \min$$

$$\text{then } \sum_{\substack{i=1 \\ i \neq e}}^{r} \frac{N_i}{\sum N_i} \operatorname{tr}_i \left(W'_k \; D'_{ik} \; D_{ik} \; W_k \right) = \min \quad \text{(Levin, 1966)}$$

So if $\frac{1}{N_e} D'_{ek} \; D_{ek} = C_{ek}$ is the experimental interval crossproduct matrix of differencescores, and the pooled cross-product matrix of the before and after interval differencescore matrix is computed as $\sum_{\substack{i=1 \\ i \neq e}}^{r} \frac{N_i}{\sum N_i} \cdot (D'_{ik} \; D_{ik}) = C_{ck}$, the problem reduces to the maximization of the ratio of two quadratic symmetric forms (Turnball and Aitken, 1932), statistically known as canonical analysis (Bartlett, 1947) and written as:

$$W'_k \left(\frac{C_{ek}}{C_{ck}} \right) W_k = \Lambda_k$$

By the double application of eigenvalue-eigenvector solution we solve this as is formulated in the following two canonical forms

$$C_{ck} = H\ Q\ H' \text{ and } H\ Q^{-\frac{1}{2}}\ H'\ C_{ek}\ H\ Q^{-\frac{1}{2}}\ H' = W_k\ \Lambda_k\ \ W_k$$

Each vector of W_k represents a change component. The change components are tested for their significance by the Wilks Λ test (Barlett, 1947) because Λ_k may be viewed as the ratio of total variance and error variance of the components. If the Chi square in

$$\chi^2 = -\left(\sum_{i=1}^{r} N_i - p - \tfrac{1}{2}\,(2m_k - p + 1) \right) \log_e \prod_{l=p+1}^{m_k} \lambda_{kl}^{-1}$$

where λ_{kl} are the elements of Λ_k in the canonical analysis, is insignificant by $(m_k-p)m_k$ degrees of freedom then the last (m_k-p) components are insignificant. Working backward from $p=m_k-1$ to $p=0$, we determine how many components are significant.

The solution of W_k is unchanged by rescaling of the original measurements to other variances, the solution is scale free, which is a nice property for interval data.
The canonical congruence loading structure of the experimental changescore factors as the congruence-coefficients between original change variables and the dimensions of the canonical analysis, follow from the linear regression formula's and standardization

$$F_k = (\text{diag}\ C_{ek})^{-\frac{1}{2}}\ C_{ek}\ W_k\ \ \Lambda_k^{-\frac{1}{2}}$$

The congruence loading matrix may be rotated by an orthogonal simple structure rotation method, such as VARIMAX (Harman, 1960)

$$F_k \; T = G_k \text{ and } T'T = I$$

The corresponding weighting matrix is computed by

$$M_k = W_k \Lambda_k^{-\frac{1}{2}} \; T$$

and the new rotated change factors are formed as the weighted sum of the difference-scores

$$D_{ik} \; M_k = Z_{ik}$$

Since $(C_{ek} - C_{ck})$ can be viewed as the matrix of real experimental change, while the other change influences, like random fluctuations, trend and so on, are estimated by C_{ck}, we may use the variance ratio's of each rotated component as the squared congruence coefficient of the change factor with the experimental condition. For the 1^{th} component this the 1^{th} element of the diagonal matrix V:

$$V = I - \text{diag } (T' \Lambda_k^{-1} T)$$

The congruence matrix of the changescorefactors is formed by the crossproduct of the partitioned matrix $Z = (Z_{eK} \vdots Z_{eM} \vdots Z_{eA} \vdots Z_{eB})$

Let r be the column vector of elements $r_{1k} = V_{1k}^{-\frac{1}{2}}$ such that

$$R = (1; r_{1K}; \; .....; \; r_{pK}; \; r_{1M}; \; .....; \; r_{pM}; \; r_{pM}; \; r_{1A}; \; .....; \; r_{1A}; \; r_{1B}; \; r_{pB})$$

Then the total congruence matrix of campaign factor, knowledge changefactors, motivation changefactors, attitude changefactors and behaviour changefactors is formed by

$$R = (r \vdots Z'Z)$$

The analysis of R is the basis for the causal inferences.

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