

THE FUNDAMENTAL DIAGRAM, A MACROSCOPIC TRAFFIC FLOW MODEL

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In models of traffic-flow, the interactions between vehicles are of prime interest, and are based on characteristics of the road, drivers and vehicles. The fundamental diagram is a representation of a relationship on a macroscopic level in the steady state between the quantity of traffic and a characteristic speed of the traffic-flow in a given road-section. The term steady state, in this connection, indicates a state of equilibrium, i.e. values are constant for a certain period of time.

The quantity of traffic is indicated by the volume ( $q$ ), or the density ( $d$ ), while speed refers to "space mean speed" ( $u$ ). The three forms of the fundamental diagram are the functions  $q(d)$ ,  $u(d)$  and  $u(q)$ ; see figure. These functions provide identical information, since  $q = d \times u$ . The exact form of the curves depends on several factors, the road characteristics, among others. However, an explanation and prediction of the variations require more detailed data and behaviour models.

Many details of design standards are derived from considerations of the interaction between a vehicle/driver combination and the road characteristics. The standards established for the road elements do not indicate the way in which they can be combined to provide an optimum design, or how will be the behaviour of traffic-flow. The fundamental diagram describes a macroscopic aspect of the behaviour, and the relationships - (as regards geometry) - will depend on factors such as cross-profile, alignment and sight distance. On the other hand, it is possible to use these relationships in road design, in the opposite sense, aiming at confining the behaviour within narrow limits, which are in accordance with the road category in question.

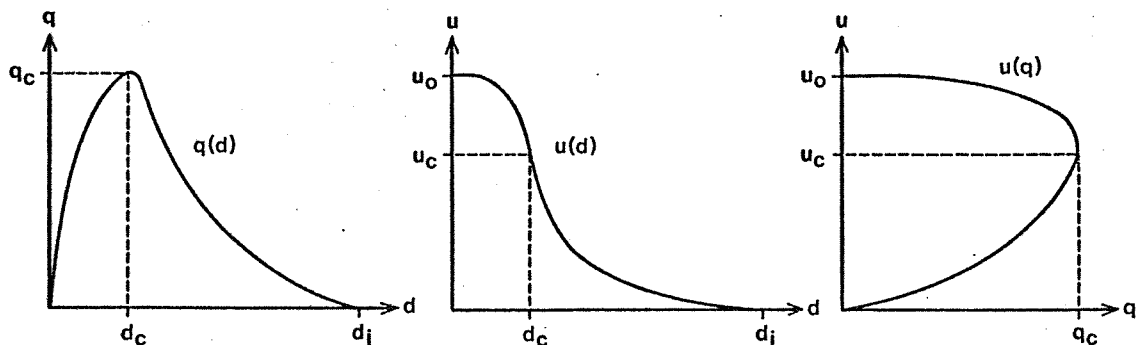
The fundamental diagram is used for establishing and determining the classification of the qualities of the traffic according to the Highway Capacity Manual (Highway Research Board, 1965). It is assumed that with a decreasing level of service, the quality for the road users deteriorates. The investigation shows, that this is not always true with regard to the aspect of safety; at low hour-volumes the accident rate on roads is relatively high, while it decreases slightly at intersections with increasing average annual daily traffic.

An inventory of the 'state of the art' of the fundamental diagram reveals, that a large number of theoretical models are available, which are based, among other things, on analogies with physical flows and the behaviour of individual vehicles. These basic conceptions, however, are not yet sufficiently evaluated and therefore the value of the results derived from them is not known. A remarkable fact is that various theories sometimes result in nearly or exactly identical fundamental diagrams. Empirical investigations have been made, mainly on motorways and the adjoining road categories. Further investigations are necessary into lower-grade roads and into the effect of factors such as rain, darkness, and discontinuous road characteristics.

The study of the fundamental diagram is simplest when the road-section is of a homogeneous character; thus, when the road characteristics are constant over the entire road-section. In that case it is sufficient to observe the road and traffic characteristics at only one cross-section. In the case of a non-homogeneous road-section, the variable road characteristics must be combined into only a few factors. Actually, this involves the quantification of the concept of friction, increased friction leads to greater effort in manoeuvring and reduced speeds. Also the observing of traffic characteristics on a non-homogeneous road-section is more complicated. Measuring methods, which seem to be applicable, are the observation of volume and vehicle identification at the ends of the road-section, and various "moving observer" methods. In the analysis, the use of the monotone function  $u(d)$  should be preferred. In addition to the mean results, an indication of scattering seems also desirable.

There is a general tendency to aim at more efficient use of existing roads; realisation of a more consistent and more easily predictable traffic behaviour for each road category and a reduction of unsafety. In order to achieve these objectives, more information is necessary concerning traffic behaviour in all its aspects, among other things, in the form of traffic-flow models. Such information can be used for the indication, design and evaluation of measures, mainly with respect to road design and traffic control. These last two measures should not be considered separately, but in combination with one another.

THE THREE FORMS OF THE FUNDAMENTAL DIAGRAM



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## 1. INTRODUCTION

In the theory and models of traffic-flow the emphasis lies on the interaction between vehicles, based on the characteristics of the road, the drivers and vehicles, and not on the interaction between a single vehicle and the road. Three levels of models, arranged in the order of increasing detail can be distinguished: macroscopic, mesoscopic and microscopic, which are concerned with mean values of traffic-flow characteristics, distribution of traffic-flow characteristics around their mean values and individual vehicle trajectories. To illustrate this, the traffic-flow characteristic speed will be considered: on macroscopic level it is the mean speed, for example, of vehicles passing a given point in a given period of time; on mesoscopic level, this characteristic indicates the speed distribution, while on microscopic level, it means the speed profiles of a number of vehicles over a road section. Before discussing the fundamental diagram - a macroscopic model - information will be given concerning the significance of the traffic-flow theory in general.

In the planning of roads, the type and design-speed depend on the function of the particular road in the general road network. Some design standards are based on models related to the behaviour of one vehicle on the road. When the road is in use, the interaction between vehicles becomes more and more important, as the quantity of traffic increases - thus determining the quality of traffic to a certain extent; (the two most important aspects of this quality are: travel time and unsafety). Traffic-flow models indicating phenomena which may occur under such circumstances (in terms of vehicle-movements) are useful, because they provide information which can result in more effective measures, both as regards road planning in a wider sense, as well as traffic control. Measures of the latter type are, as a rule, regarded as an addition to a road design which is no longer adequate. It would seem more effective to take the possibilities of traffic control into account at the planning stage of the road. As an example of this, the traffic-flow model constructed for a part of the motorway network around San Francisco, should be mentioned (see May, 1974), which provided an integrated simulation of the

effect of improvements in roads and ramp control.

The fundamental diagram is a macroscopic traffic-flow model representing the relationship between volume, density and mean speed. These relationships are not fixed, but depend e.g. on road characteristics. The interpretation and the forecasting of variations requires more detailed traffic-flow models, as well as data concerning the behaviour (perception, information processing, decision and manoeuvring) of the driver.

## 2. DEFINITION

The fundamental diagram is a representation of a relationship, that exists in the steady-state, between the quantity of traffic and a characteristic speed of traffic on a road section. The term "steady-state" means, in this connection, that an equilibrium is considered, i.e. relevant values are constant during a certain period of time. The quantity of traffic may indicate the number of vehicles passing over a given point during a given period of time, the volume, or the number of vehicles present on a given road section at a given moment, the density. In connection with speed we have to consider the mean speed over the road section ("space mean speed"), as well as the operating speed (i.e. the mean speed over the road section which can still be maintained with safety over that road section, under the prevailing traffic conditions). The determination of the operating speed is not a simple matter, it may be assumed, however, that it corresponds to the 85% point of the speed distribution (cf. for example Rørbech, 1972). The mean speed has been chosen here, because it is defined more specifically. At the same time a direct relationship can be established between various forms of the fundamental diagram, since volume equals density times "space mean speed". For the sake of simplicity, in the following part, "space mean speed" will be simply called "speed".

Volume will be indicated by "q", density by "d" and speed by "u". The three forms of the fundamental diagram are the functions  $q(d)$ ,  $u(d)$  and  $u(q)$ . They are shown schematically in Fig. 1 and all three contain essentially the same information. The form  $u(q)$  is used most often and, as a rule, only the upper branch. From a theoretical point of view, and sometimes also from practical points of view, it is a drawback that it is a two-valued function. Volume alone is therefore an unsuitable characteristic to represent the state of the traffic-flow. From the remaining two, density is to be preferred because the function  $u(d)$ , and consequently  $u(q)$ , is often horizontal for low densities. In that case density determines speed but not vice versa.

The quantitative trend of the curves may depend on the following factors:

- road characteristics, such as geometry and speed limits
- vehicle characteristics, e.g. vehicle type
- driver characteristics, e.g. the purpose of travel
- other circumstances, such as rain and darkness.

Salient points of the diagrams are:

- the free speed  $u_0$ , the speed at which  $q = 0$  and  $d = 0$
- the capacity  $q_c$ , the maximum value of  $q$
- the critical density  $d_c$ , the density at which  $q = q_c$
- the critical speed  $u_c$ , the speed at which  $q = q_c$
- the "jam" density  $d_j$ , the density at which  $u = 0$  and  $q = 0$ .

In a certain way the curves are upper limits of average driver capacities, which may be exceeded incidentally. Points inside the curves correspond to lower speeds for a given density; or bigger following-times and distances for a given speed, than normally occur.

The fact that, of the three variables: volume, density and speed, the first relates to a fixed point along a road and a period of time, while the other two relate to a road section and a moment of time, may cause complications. Edie (1965) established definitions, in which all three variables relate to a road section and a period of time:

$$q = \sum x_i / XT \quad d = \sum t_i / XT \quad \text{and} \quad u = q/d = \sum x_i / \sum t_i$$

Summing is effected for all vehicles in the considered space-time domain of a size  $X$  times  $T$ ;  $x_i$  is the distance covered and  $t_i$  the period of time taken by the vehicle  $i$ ; (see Fig. 2).

Actually, only the moments of time or the positions at the edges of the domain under consideration, are necessary for carrying out the summations; the trajectory and identification of the vehicle can be dispensed with. Although the variables defined in this manner cannot easily be observed, they provide a theoretical basis; and with the aid of these variables it is possible to establish the fundamental diagram of a road network.

Brilon (1974) observed volumes and densities locally (local density is the quotient of volume and harmonic mean of local speeds) and for a space-time domain on a rural two-lane road. He found a satisfactory agreement, which was to be expected, since the road section had a homogeneous character, and the traffic-flow was, most probably, in the steady state.

More or less opposed to the approach of Edie, there is a procedure according to which all variables are defined and observed locally. In this case density is replaced by "occupancy" (the percentage of time during

which a cross-section of the road is occupied by a vehicle), which can be measured fairly easily. Under steady state conditions and in the case of vehicles having the same length, the degree of occupancy is in direct proportion to the density.

### 3. IMPORTANCE FOR ROAD DESIGN AND TRAFFIC QUALITY

In road design, the function of the road in the road network under consideration, the volume of traffic to be expected and the requirements for the quality of traffic operation, are the starting points, which are, to some extent, inter-related. Many details of the design standards are derived from considerations concerning the interactions between a vehicle/driver combination and the road characteristics. Thus, standards for radii of curvature and superelevation (for example) are established, which are dependent on the design speed. The requirements of the various road elements do not provide absolute guide lines for the way in which the road must be made up of these elements, in order to obtain an optimum design. For example, when choosing their speed, drivers, as a rule, are influenced more strongly by the general aspects of the road and its surroundings, than by the separate road elements, (see for example Janssen, 1976). Moreover, it is not yet known, how traffic behaviour will be influenced, if not only the interactions between individual vehicles and the road, but also the interactions between vehicles are considered. Certain aspects of such behaviour are described by traffic-flow models and the fundamental diagram, which establishes relationships between mean traffic-flow characteristics, can be considered as a first step. Such macroscopic relationships are dependent on road characteristics, as regards the geometry on cross-section, alignment and the sight distance. On the other hand, such relationships can be used for designing roads with the object of realising a certain specific traffic-flow behaviour. This aspect is becoming increasingly important, since at the present time, the tendency to confine this behaviour within narrow limits (corresponding to a certain road category) is much greater than in the past.

Volume in relation to capacity and speed, (either operating speed or mean speed), has a considerable effect on the quality of the traffic-flow. In the Highway Capacity Manual of the Highway Research Board (1965) such an influence has been practically achieved in the form of a number of levels of service; denoted by A, B, C, D, E, F, in order of decreasing quality (see Fig. 3). When applying this concept, the relationship between operating speed and volume capacity ratio is used, depending on the road characteristics. It is assumed, that, from the driver's point of view, the level of service comprises the following aspects of quality: travel time; traffic interruptions or restrictions; freedom to manoeuvre; safety; driving comfort and convenience; vehicle operating costs. The way in which these

various aspects are combined into one, is not known. In general, it can be assumed that the part-aspects are reduced with reducing levels of service, with the exception of safety, which must be investigated more thoroughly.

Dutch investigations have shown that, on motorways, the accident rate (number of accidents per vehicle mileage) increases with decreasing levels of service (see Beukers, 1974); while for single-carriageway rural roads a "U-shaped" relationship exists, with an optimum level of service "C". The representative level of service has been established on the basis of the 30th busiest hour of the year.

More detailed investigations have been carried out for all the hourvalues in a certain period (Gwynn, 1966 and 1970; Leutzbach, 1970 and 1973), establishing a U-shaped relationship between volume and accident rate on rural roads of various types. For levels of service A - E, the volume is, in most cases, a reliable criterion. In cities, intersections have mainly been investigated. On the basis of a large amount of data, Leong (1973) found that the accident rate (number of accidents per number of passing vehicles) has a constant or slightly decreasing value, with increasing average annual daily traffic. Considerably less is known about rural intersections and urban arterials; this results from their relatively lower proportion of accidents.

From all these considerations it can be concluded, that the level of service is not always a reliable criterion for the aspect "safety".

Among other things, due to financial limitations, there is a tendency to increase design volumes (see for example T.E.C. 1974). As a result of the long-term tendency for increasing speeds, and a supposedly flattening of the upper portion of the speed-volume curve, it can be assumed that travel times will not increase greatly. In view of the above statement a negative effect on safety can be expected. Should this indeed be true, traffic-control systems must be recommended, which are capable of counter-acting, the combination of a low level of service and a high unsafety. In the assessment of various aspects of quality (also involving the effects of the traffic-flow on the environment), it is recommended, that a generalised cost/benefit method should be used (see for example Flury, 1976).

#### 4. SOME APPLICATIONS

##### Design of the road system

On laying out a future optimum rural road system Nederlands Economisch Instituut (1972) takes into account: travel time costs; exploitation costs of travelling by car; accident-costs and investments and maintenance costs of the road. In this respect, use is made of the fundamental diagram and the relationship between accident rate and levels of service.



Minimal social expenses occurred in connection with the increased load during the evening rush-hour, which was higher than the standard established in the Netherlands. In view of the uncertainties in the assumptions made, it has been decided to use the existing standard, i.e. level of service C.

#### Design of measures

Improving the traffic-flow is sometimes possible by influencing the choice of the route (distribution), and limiting the passage of traffic (dosage) at given places. The design of such measures requires, among other things, the fundamental diagrams of the respective arteries (see for example, Stock, 1973).

#### Evaluation of measures

Zackor (1972) studied the fundamental diagram for the evaluation of traffic-dependent speedlimits on motorways. It was found that by controlling the speed, a greater capacity was obtained, with higher density and mean speed

#### Fundamental diagram for more detailed models

The fundamental diagram indicates a relationship in steady-state between macroscopic traffic-flow characteristics. On considering dynamic situations such as the kinematic and shock waves of Lighthill (1955) or the more recent model of Payne (1971), it can be used as a state of equilibrium.

### 5. STATE OF THE ART

#### 5.1. Theoretical aspects

There are many mathematical models for the fundamental diagram, based on general considerations, analogy with physical flow, and the behaviour of individual vehicles. The following should be mentioned:

- Deformation of the free speed distribution

Haight (1963) assumes that the speed distribution gradually changes from zero density, to complete jamming, becoming more and more limited in upwards direction. With the aid of an arbitrary choice of the theoretical speed distribution and the boundary curve, a fundamental diagram can be derived, which, in principle, is valid for all densities.

- Analogy between traffic-flow and a flowing compressible medium

Volume, density and the speed of traffic flow, are regarded as continuous values, which can be differentiated. A conservation of the vehicles is assumed, the existence of a fundamental diagram and a relationship between the derivative of speed with respect to time and that of density with respect to space. Based on these assumptions it is possible to derive the form of the fundamental diagram; (for example, see Drew, 1965). The third

assumption is only valid above a certain density.

- Analogy between traffic-flow and a flow of particles.

Based on techniques used in statistical mechanics, an equation, similar to that of Boltzmann, is established for the speed distribution of vehicles as a function of space and time, (see Prigogine, 1971). Given the free speed distribution, a special solution of this equation yields a fundamental diagram, which, in principle, covers all densities.

- Keeping distance in a static way.

In heavy traffic, the drivers will keep a certain distance from the preceding vehicle, this distance being assumed to depend on the reaction time, the speed and the estimated braking capacity of the leading as well as their own vehicle. Additionally, it is assumed that the entire traffic-flow can be represented by a vehicle displaying average behaviour. Based on these assumptions a fundamental diagram is obtained, in which the microscopic distance and speed, are replaced by macroscopic density and mean speed, such diagram being valid above a certain density.

- Keeping distance in a dynamic way.

A dynamic interpretation of the follow-the-leader behaviour in heavy traffic is given by 'car-following' models. As in the preceding case, the fundamental diagram is derived by a transition from microscopic variables to macroscopic variables, (see for example, Gazis, 1961 and Ceder, 1974).

The above-mentioned models are the most important examples of a steadily growing collection of models. None of the theories, on which these diagrams are based, have been sufficiently evaluated, and the results should be treated accordingly, and no priority can be established. It is remarkable, that different theories sometimes result in very similar, even identical diagrams.

## 5.2. Empirical aspects

The empirical information available at present, can be summarised as follows:

- The important parameter: capacity, has been thoroughly studied. The Highway Capacity Manual (H.C.M.) determines ideal values for various types of road, and gives information about the deviations, which occur in the case of non-ideal road and traffic characteristics.
- There is rather much information, relating to motorways, concerning diagrams for densities, less than the critical density, (see for example H.C.M. and Beckmann, 1973).
- There is less information relating to other roads. In this case it is also more difficult to obtain general information, due to the larger variations in road characteristics which - by one means or another - must be reduced to as few parameters as possible. This problem will also be discussed in the section concerning investigation aspects.

- The effect of the other factors, such as rain and darkness, has received little investigation.
- As an exception, the investigation of Jones, (1969), into the effect of rain on the capacity of a motorway, should be mentioned. (N.B. When designing road elements (for example bends), the designers, as a rule, take wet road surfaces for the basis).
- As a special case, the situation arising at a bottle-neck should be mentioned, which sometimes involves a discontinuity in volume and speed in the surrounding of the critical density. However, the situation is not yet fully understood. (see Edie, 1961 and Beckmann).
  - Discontinuities can also occur on main roads in the same density region (see for example Ceder and May, 1974).
  - Universally applicable mathematical models have not yet been established. Linear and quadratic relationships, are, as a rule, applicable in the range leading up to the critical density.

The above statements show that there are still gaps in our knowledge relating to the fundamental diagram. In addition, some results may become obsolete, due to changes in vehicle and driver characteristics and control measures, (for example speed limits).

The detailed data contained in the H.C.M. cannot always be applied outside the United States, due to differences in types of cars, driver behaviour and other factors.

Follow-up investigations can be carried out in an expedient manner by categorising the roads (Janssen). The organisation of the investigation will be discussed in the following part, since the investigation methods are important with regard to the interpretation of the results.

## 6. ASPECTS OF INVESTIGATION

### 6.1. General aspects

The investigation consists of collecting data, i.e. characteristics of road and traffic and circumstances, as well as the analysis of this data. An important factor in the choice of the road section is whether a part of the fundamental diagram, or the complete diagram, should be established; in the latter case, it is necessary that congestions occur in regular or predictable intervals of time.

Another important factor is, whether the road sector under investigation, is homogeneous or not; (i.e. whether the road characteristics are invariable over the entire test section). In the case of homogeneous road sections it is sufficient to take only at one cross-section the road and traffic characteristics. However, the investigation of non-homogeneous road-sections is more complicated. The variable road characteristics must preferably be

reduced to very few parameters. For example:

- H.C.M. and Rörbech (1972) used the percentage of the roadlength having sight-distance greater than 450m, which combines alignment and lateral clearance.
- Duncan (1974) makes a distinction between hilliness (average derivative of the vertical profile) and bendiness (average derivative of the horizontal profile);
- Rankine (1974) established parameters representing the effect of parked vehicles, intersections and the environment, (in terms of land use) on urban roads.

Actually all these parameters imply a quantification of the 'friction' concept; greater friction results in increased manoeuvring efforts and lower speeds.

The collection of traffic characteristics is also more difficult for non-homogeneous roads. The volume can still be observed at one cross-section, but not the speeds.

The circumstances which must be taken into consideration include the day of the week and the time of the day (defining, in general, the purpose of travel), as well as the conditions of light and the weather.

## 6.2. Measuring methods

The choice of a measuring method depends on the purpose and the size of the investigation, as well as the available means, in terms of money, man power, and measuring and processing equipment. In assessing a measuring method, or, more exactly, an observation method, which consists of measuring, registration and processing, the following aspects must be considered:

- Conspicuousness (by which the behaviour to be measured may be influenced);
- adaptation of measuring, registration and processing;
- limitations of application;
- reliability and accuracy;
- ease of installation;
- costs.

It is beyond the scope of the present paper to assess and compare all measuring methods, taking into account the above aspects. We shall confine ourselves to a short description and the most important characteristics of some measuring methods, without taking into consideration measuring instruments.

### Observations on a single road cross-section

Volume and individual speeds are measured. In view of mean speed it is

recommended that the harmonic mean value should be used instead of the arithmetic mean value, because the former gives a better approximation to the "space mean value". Density can be assessed by the quotient of volume and mean speed. Instead of density the occupancy can also be used. The vehicle type can be determined by visual observation or by photography or video apparatus, or it can be derived from actual vehicle characteristics measurements (length, height, weight). The method can only be applied to homogeneous road-sections; in other respects, the characteristics depend on the instruments used.

- Observations on two road cross-sections

a) Travel time being obtained by means of vehicle identification. The volumes, moments of passage and identification of vehicles are registered at each end of a road-section; vehicle identification takes place by visual observation or by photography. The actual travel times of the vehicles are calculated from the difference in the moments of passage between the ends. This method requires complicated processing, since the data obtained for each road cross-section must be combined.

Note: The above two methods actually supply more data than is necessary for plotting the fundamental diagram: in addition to mean speed and travel time, they also reveal individual values.

b) Travel time, with correlation of the volume pattern, (Wright, 1974). This method is based on the assumption that variations in volume, travel at the same speed as the vehicles. According to the traffic-flow theory of Lighthill, however, this is only valid if the speed equals the free speed. Wright introduces less restricting limitations i.e. no congestions and no controlled intersections on the road-section. He finds the method suitable for the automatic registration of volume. The actual practical value of his method still has to be determined, while the above theoretical limitations are actually present.

- "Moving observer"

On a road with traffic in both directions, the number of vehicles overtaking and being overtaken, the number of oncoming vehicles, and the travel time of the vehicle itself, are observed from a vehicle, moving with the traffic-flow. These observations must be made while travelling in each direction; but on roads with one-way traffic the observations must be made during two trips, having a different travel time. Based on the observed number of vehicles and travel times, it is possible to assess the volume and speed of the traffic-flow. The method is simple, inexpensive, and does not supply more data than necessary.

- Combination of stationary and moving observers

a) "Floating car"

Volume is determined at one or more cross-sections of the roadsection under investigation, while the speed of the traffic-flow is measured by the travel time of "moving observers", who attempt to drive at the mean speed.

The latter measurement introduces a subjective element into the investigation method. The characteristics of the method depend on the instrument used in measuring the volume and have a certain similarity with those of the "moving observer" method.

b) "Closed Vehicle Trajectory" (Wright, 1973, see Fig. 4).

Here traffic is observed in one direction only. Two moving observers drive off from a point within the road-section, towards each end of the section; one of the observers records the number of oncoming cars, while the other observes the number of cars overtaking and being overtaken. These observations yield an initial value of the number of vehicles on the road-section. Next, volumes are measured in short units of time, at the section ends, and on this basis, volume and density (as defined by Edie) can be assessed. Next the trajectory is closed by carrying out the first procedure, in the opposite direction, thus providing possibilities for correcting the effect of counting errors. The practical value of this method has still to be verified. A limitation of the method is the fact that no other cars may enter or leave the test section; and a drawback is the considerable effect of counting errors, which may be present, in spite of the corrections.

#### - Aerial photography

The entire road-section is photographed (or filmed) from above repeatedly. Thus, as opposed to others, the method can be regarded as continuous, with regard to both time and space. After processing the data into vehicle trajectories, the volume, density and mean speeds can be obtained by various methods including those discussed above. However, this data can also be obtained by a simpler method; (c.f. "closed vehicle trajectory" method). The method is very expensive, due to the amount of work required for reading the films, and it can only be used under favourable conditions of visibility.

### 6.3. Analysis

#### - Period

Traffic data is generally determined for a certain period of time; from this, the question of, how long this period should be, arises. The fundamental diagram represents a relationship in a state of equilibrium; and strictly speaking, the optimum period is that which lasts as long as the equilibrium state. In practice, a certain predetermined period is chosen, which is sufficiently short so that, in general, no large change could have occurred in the equilibrium state; while, on the other hand, it is sufficiently long, so that the fluctuations in the random samples do not become too great. A quantitative assessment has been made by Zackor (1972), yielding 5 minutes as an adequate value for motorways. When using too long a period, convex functions, (such as  $q(d)$  and  $u(q)$ ) can be unjustifiably flattened. For this reason the use of the monotone fundamental diagram  $u(d)$  should be preferred in the analysis, even if the densities have to be calculated for

this purpose; afterwards the other relationship can be derived therefrom.

- Distribution

As a rule, the aim of the investigation is the determination of a dependent variable, such as the mean speed, on the basis of independent variables, such as volume or density and road characteristics. Such independent variables, in general, are not spread evenly over the area under investigation; and on account of this, frequently occurring values may have a disproportionate influence on the results. This can be prevented by selecting a group with uniform distribution, from among the variables collected.

- Model

In general, it is desirable to represent the data collected as a mathematical model. Such model can be chosen from the above selection of theoretical models, or it can be based on a more empirical background. The model must meet the following requirements: it must be in accordance with the data; it must be flexible (sufficient parameters); it must be simple (preferably with simple functions and parameters, which can easily be estimated); and it must have parameters which can be interpreted in traffic terms. Linear regression techniques (sometimes after transformation into a linear model) and non-linear regression techniques, are suitable methods for adjusting the model to the data. At the same time an indication can be given as to the possible scattering of the dependent variable, (for example in the form of a confidence interval). This is a very useful adjunct for the interpretation of a result, but is, however, only seldom used.

## 7. DISCUSSION

It is desirable to:

- make better use of the existing infrastructure;
- establish a more consistent and more easily predictable traffic behaviour for road categories;
- reduce traffic unsafety.

In order to realise these objectives, more information must be obtained about the aspects of traffic behaviour, and especially the analysis of the driving task, the interaction between vehicles and the general traffic-flow characteristics, (as a function of road characteristics, measures and circumstances). The fundamental diagram presented, is intended as a contribution towards the solution of these problems, and can be applied to the design of roads and to designing measures for controlling the traffic. However, the diagram is not yet sufficiently understood and requires more detailed study.

Further investigations are required into the relationship between the levels of service, which are connected with the fundamental diagram, and the quality of traffic, and primarily into the aspects of traffic safety. It must be taken into account that the various relationships are not invariable, but can be modified by measures, relating to people (training), vehicles, road characteristics and regulations including traffic control.

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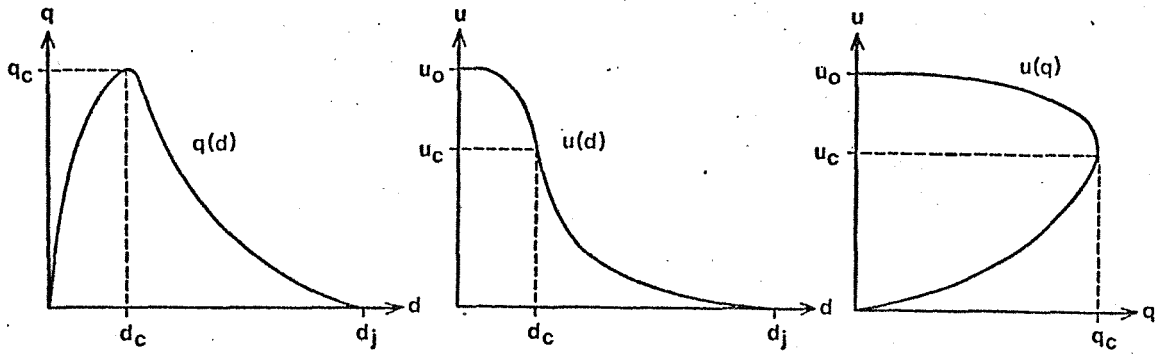


Figure 1. The three forms of the fundamental diagram

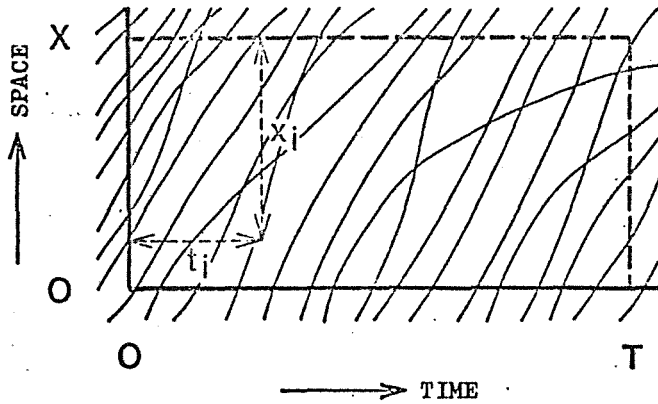


Figure 2. Definitions for a space-time domain

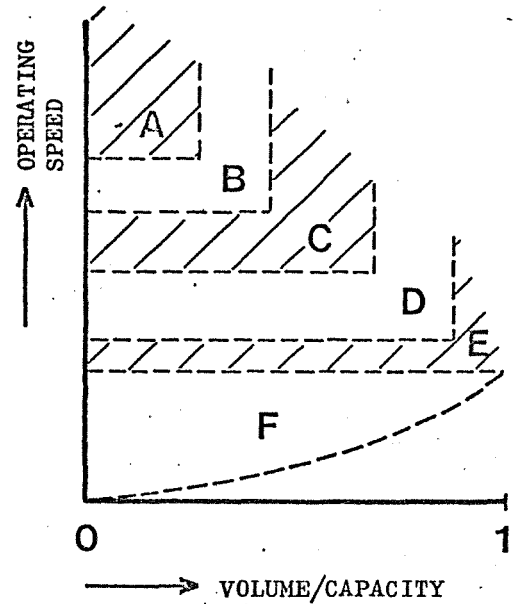


Figure 3. General concept of levels of service (from Highway Capacity Manual, 1965)

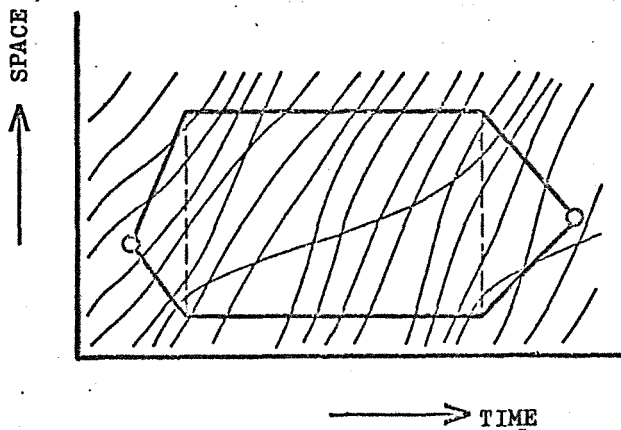


Figure 4. Measuring method "Closed Vehicle-Trajectory"