DEVELOPMENT OF ROAD SAFETY IN SOME EUROPEAN COUNTRIES AND THE USA

A theoretical and quantitative mathematical analysis

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The mathematically non-interested reader is advised to skip the parts marked by points in the margin of the text. Graphical presentations will explain these parts of the theory.

ABSTRACT

The development of traffic and traffic safety over long periods is viewed as long-term change in system structure and output in the context of self-organizing and learning systems. The theoretical analysis states that society

a.- creates changes in the road traffic system in order to accomplish more positive outcomes

b.- adapt the system to negative outcomes of these changes

c.- stabilize the system at satisfaction level.

Relevant changes in the traffic system are foremost expressed by growth of traffic volume as a result of road enlargement and growth of the number of vehicles and distances travelled. On the basis of supply-demand considerations, mathematical models for traffic growth are proposed. Growth of traffic volume leads to growth of exposure. The relation between traffic volume and exposure is mathematically constrained by a power-transformation of volume to exposure.

Growth of exposure in a partial-adapted traffic system leads to negative outcomes, e.g. accidents. Risk reduction is viewed as adaptation of the system and is described in terms of mathematical learning theory. It is conjectured on theoretical grounds and empirically demonstrated by data from several countries, that the long-term development of the number of fatalities is not a function of the level of traffic volume but of increment in traffic volume. Since fatalities result from insufficient adaptation of the system, the reduction of fatality risk as an adaptive process may tend to nearly zero at the time the traffic system has approached the level of saturation of traffic volume. The development of outcomes between the continuum of expected encounters (pure exposure) and fatalities, like conflicts, damage only accidents and injuries, is on theoretical grounds mathematically described as a weighted sum of exposure (= function of traffic volume) and fatalities (= function of changes in traffic volume) and consequently will not reduce to zero at the end of the growth of the system. Data from several countries illustrate the validity of the theory. Results confirm the postulated mathematical relation between the development of increments in traffic growth and the development in traffic safety. A basic comparison of the development for several countries in Europe and the USA is given by analysis of the data.

CONTENTS

1. INTRODUCTION

The development of traffic and road safety over long periods of time is described by several authors (Appel, 1982; Blokpoel, 1982; Brühning et al., 1986; Koornstra, 1987; Minter, 1987; Oppe, 1987; Oppe et al., 1988; Haight, 1988) as related processes resulting in a steadily decreasing fatality rate. Blokpoel, Appel, Brühning and Haight use linear approximations for either growth of traffic volume or fatality rate or both, whereas Oppe, Minter and Koornstra use non-linear functions for growth of traffic (sigmoid growth curves) and non-linear decreasing functions for the fatality rate (log-linear or logistic curves).

Apart from limit constraints (non-negative number of fatalities) and mathematical elegance, no theoretical justifications for these linear or non-linear functions are given. Oppe refers to a saturation assumption for the choice of symmetric sigmoid curves for traffic growth. Minter implicitly makes similar assumptions, but also refers explicitly to learning theory for the justification of the fatality-rate curve, as did Koornstra. Comparing these applications with standard knowledge in mathematical psychology (see Sternberg, 1967), Koornstra applies the linear-operator learning model (constant reduction of error probability) and Minter the so-called beta-learning model (reduction of error probability as a logistic decreasing function). All authors, except Minter for the fatality rate, describe these functions with time as the independent variable, whereas mathematical learning theory takes the number of relevant events as explanatory variable.

Perhaps the most remarkable result is presented by Oppe (1987), where he demonstrates that the parameters of the fatality-rate curve are empirically related to the parameters of the growth curve for traffic volume. Koornstra (in: Oppe et al., 1988) proves that this relation of parameters allows the number of fatalities to be a function of the derivative of the function for traffic growth. One may wonder why fatalities seem to be related to the increase of traffic volume and not to the level of traffic volume itself. Clearly, some theoretical reflection is in place.

2. GENERAL SYSTEMS APPROACH

At an aggregate level and over a long period of time one may view traffic and traffic safety as long-term changes in system structure and output. Renewal of vehicles, enlargement and reconstruction of roads, enlargement and renewal of the population of licensed drivers, changing legislation and enforcement practices and last but not least changing social norms in industrial societies are complex phenomena in a multi-faceted and interconnected changing network of subsystems within a total traffic system. The steadily decreasing fatality rate can be viewed as adaptation of the system as a whole to accommodate and evade the negative outcomes.

2.1. Evolutionary systems

The above-mentioned characterization of the system can be compared with evolutionary systems, known as self-organizing systems (Jantsch, 1980) in the framework of general-systems theory (Laszlo et al., 1974) .
There are striking parallels between the growth of traffic and the growth of a population of a new species. In Figure 1 we picture the main elements of such an evolutionary system in population biology.

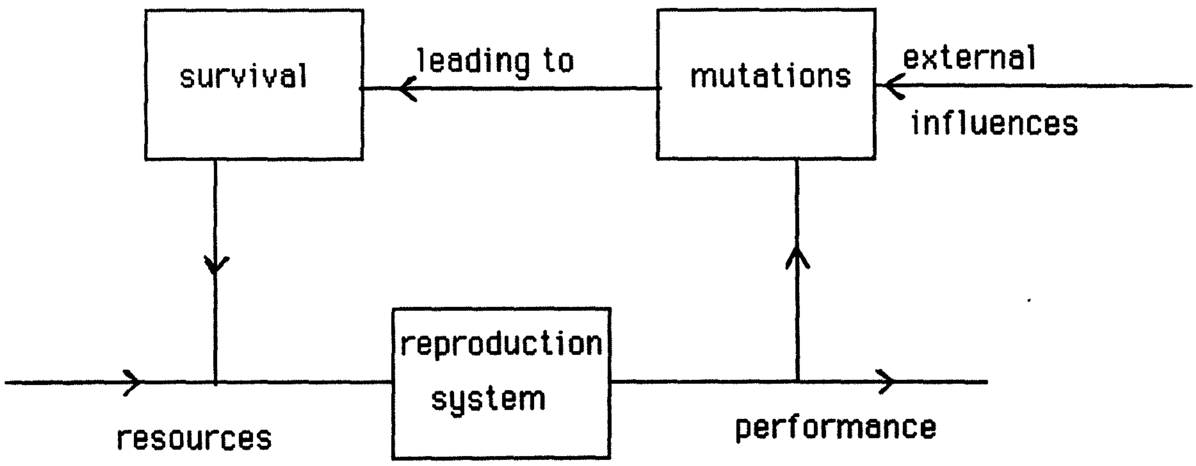

Figure 1. A model of a biological system.

Mutations are the basis for the formation of new aspects of functioning in specimen of an existing species. The survival process by selection of the fittest, leads to a reproduction process of those elements which are well adapted to the environment. The result is an emerging population of the new type of the species. The process of selection and reproduction guarantees that only those members who survive the premature period, will produce new-offspring. The selection process leads to a growing birth rate as well as to a reduction of probability of non-survival before the mature reproductive life period. The resulting growth of a population and the development of the number of premature non-survivors is pictured in Figure 2.

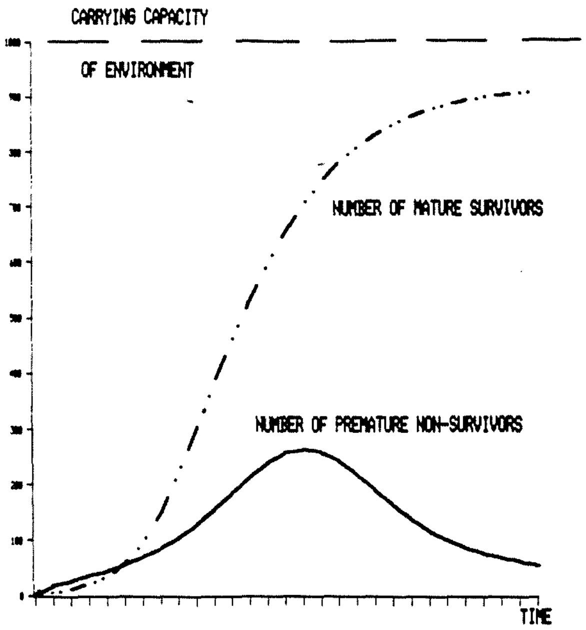

Figure 2. Evolution of a population.

Our main interest in this process is the rise and fall of the number of premature non-survivors. The growth of new-born members in the population follows a lower S-shaped sigmoid curve similar to the growth of the population. In combination with a steadily decreasing probability of death before mature age, this results in the bell-shaped curve of the number of premature non-survivors. Under suitable mathematical expressions, used in population biology (Maynard Smith, 1968) such as logistic equations, this bell-shaped curve can be mathematically described as proportional to the derivative of the growth equation. The generalized assumption of this notion could be formulated as follows:

- the development of the number of negative (self-threatening) outcomes of a self-organizing adaptive system is related in a simple mathematical way to the development of increase for positive outcomes-.

Looking upon the traffic system as a self-organizing adaptive system it is tempting to translate this conjecture as:

- the development of the number of fatal traffic accidents per year is in a simple mathematical way related to the yearly increment in traffic growth-.

2.2. Open and closed systems

The differences between open input-output controlled systems and closed self-organizing adaptive system, however, must be well understood in order to judge the validity of such analogy from biological systems to social, technical or economic systems. In Figure 3 a diagram of an open management system (taken from Jenkins, 1979) is given.

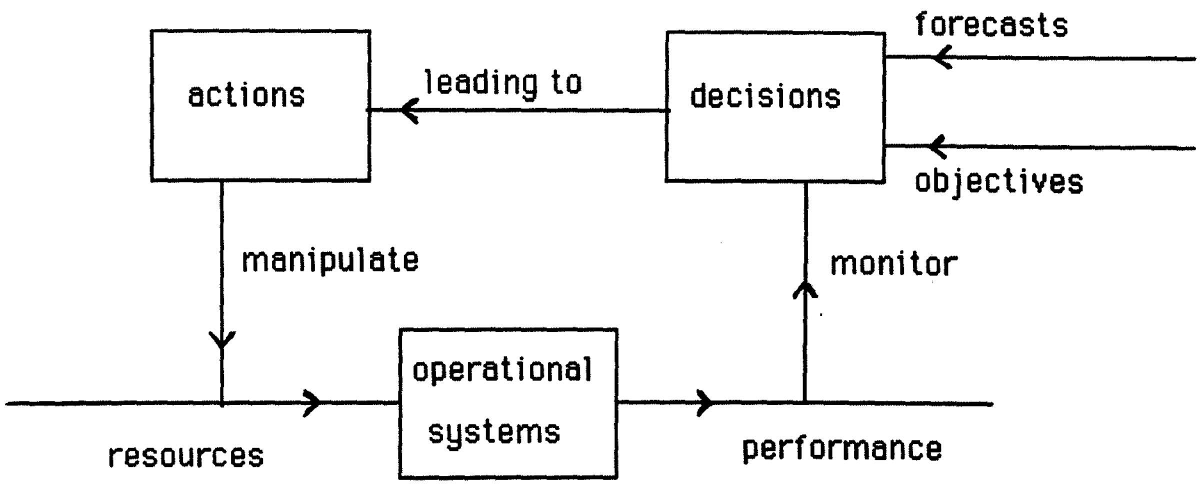

Figure 3. A model of an open system of management.

In such open systems feedback goes from output to input through a comparator based on extrapolations and objectives. Unlike biological systems, here this process is not governed by an automatic or blind mechanism like mutation, but by actions of a deliberate decision-making body. The control is directed to manipulation of the input resources by actions of individuals, collective bodies or even other subsystems of a more or less physical nature. The system is called an open system, since the feedback is a recursive relation between output to and input from the environment, while the inner operational production subsystem itself is unchanged.

In contrast to such an open system, we may picture an even more relevant "closed" system of management as is given in Figure 4.

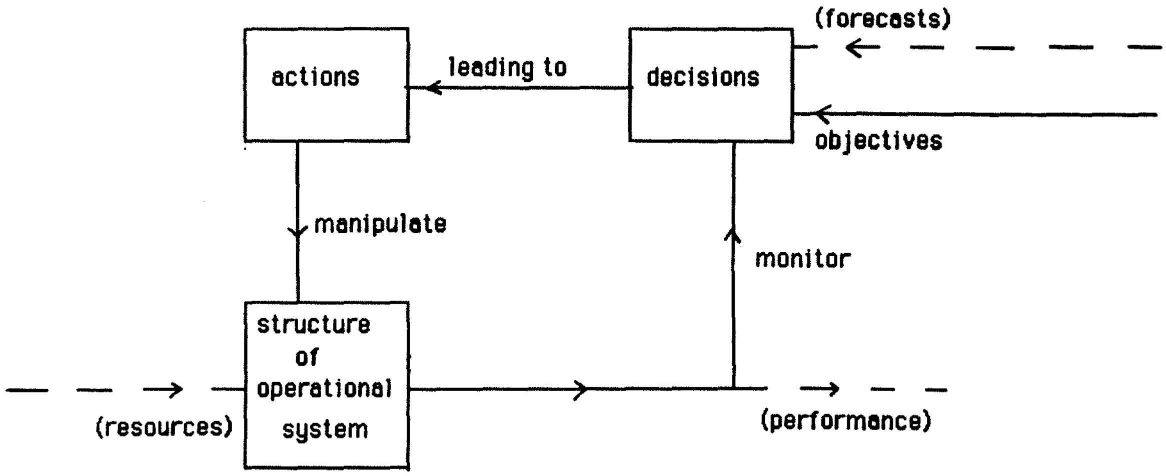

Figure 4. A model of a "closed" system of management.

Here the recursive loop in the system is hardly based on input-output relations. Again the comparator is a decision-making body. It compares intermediate output with given objectives, but now the action leaves the input unchanged as a given set of resources and changes the structure of the operational production process in order to bring the output performance in accordance with the objectives. The system is called a closed system since it operates within the system by changes in the substructure of itself. It takes the outside world from which the input comes as given and does not control the input. The effects of output are mainly viewed as intermediate and directed to the inner parts of the system.

The close resemblance to the biological system of Figure 1 is apparent. Now instead of a blind mutation and selection process we have deliberate actions from a rational decision-making body, but the structure is more or less identical with respect to its closing. This closing is even stronger in the diagram of the closed management system. Resources or necessary energy use of the system are taken for granted, although the environment of the closed system is a crucial condition for the existence of such systems. But given the environmental boundary conditions for the system, its functioning within these boundaries can be analyzed as internal throughput production without regard to manipulation of the given input.

In classical open systems the mathematical description is based on matrices or vectors for input and output related by transformation matrices, which correspond to the working structure of the system and are generally expressed by linear algebraic equations (Desoer, 1970). The aim of control in this type of system is the maintenance of stability at a (desired) equilibrium level of output through manipulating the input.

In closed systems the input is not manipulated and instead of transforming the input, the transformations of the input themselves change, since the operational structure itself is changing. Due to its changing operational structure the mathematical description of closed systems is quite problematic.
In general, closed systems are self-referencing systems where output becomes input. They are concerned with intermediate throughput instead of input and output, and generally handle development of throughput in non-equilibrium phases of the system. The development of throughput is foremost described by non-linear equations, like throughput equations in electrical circuits as a classical closed system or throughput equations in catalytic reaction cycles in modern chemical closed systems (see Nicolis & Prigogine, 1977). Except in these cases of complete self-reference where the output is the only source of relevant later input and where change is autonomic, so-called autopoietic systems (see Varela, 1979; Zeleny, 1980), the field of closed systems is far less developed in a mathematical sense.
However, for most social systems the relevance of closed systems is much larger, than open systems. Every change of law, every reorganization of a firm, every new machine in a factory is a change in the operational structure in order to enhance the quality and/or quantity of the performance, but cannot be analyzed by the classical control in equilibrium systems.

Except the universe itself, a system is never closed, nor solely an open system, perhaps excluded man-made technical production systems. Most complex real-life systems can be described as both open and closed. The simultaneous mathematical description, however, is generally still intractable. Although such systems are mathematically difficult, on a conceptual level they can easily be described simultaneously and as such are pictured in the diagram of Figure 5 (taken from Laszlo et al., 1974).

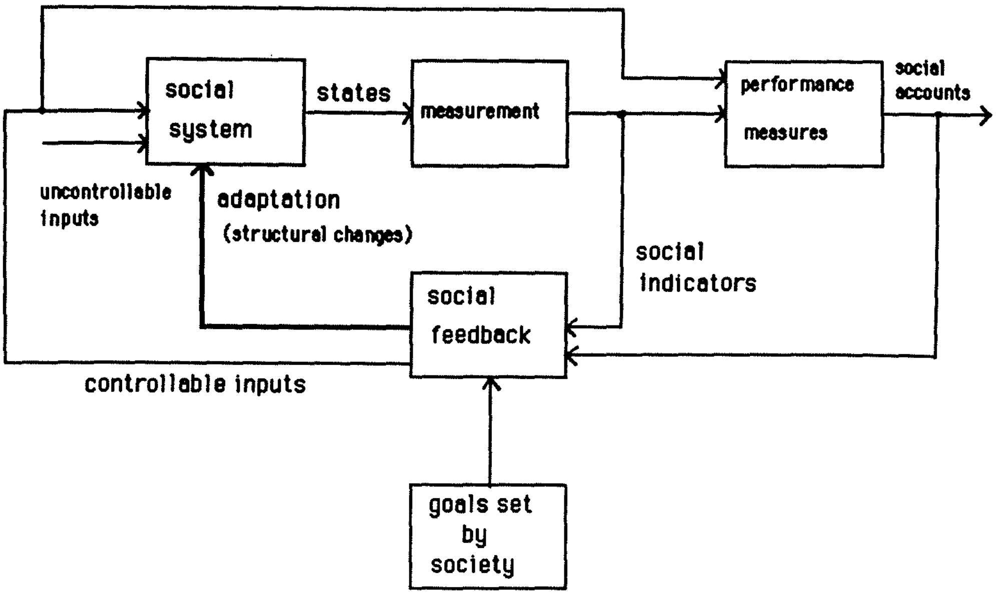

Figure 5. A model of an open and "closed" social system.

We apply this social-system description to the emergence of motorized traffic and traffic accidents. We concentrate on the inner closed feedback loop from measurement of performance through the feedback compartment to structural changes in the system as an adaptation process on a conceptual level. Subsequently the quantification of the development of throughput in the system is mathematically analyzed.

2.3. The "closed" traffic system

The emergence of traffic and traffic accidents can be described as a closed system in the following way. Society invents improvements and new ways of transport in order to fulfil the need of mobility of persons and the need of supply of goods. These needs and objectives are mainly met by the development and increasing use of cars and roads in modern industrial society.

This is done by

- building roads, enlarging and improving the network of roads,
- manufacturing cars and other motorized vehicles, improving the quality of vehicles and renewing them and enlarging the market of buyers of these vehicles,
- teaching a growing population of drivers to drive these cars or other motorized vehicles in a more controlled way for which laws are developed and enforcement and education practices are improved.

This growth and renewal can be quantified by numbers of car owners and license holders, by length of roads of different types and as a gross-result by the fast growing number of vehicle kilometers. We take vehicle kilometers as the main indicator of this growing motorization process of industrial society.

The negative aspect of this motorization is the emergence of traffic accidents; as an indicator we may take the number of fatalities. The adaptation process with regard to this negative aspect can be described as increasing safety per distance travelled, made possible by the enhanced safety of roads, cars, drivers and rules. Reconstructed and new roads are generally safer than existing roads, new vehicles are designed to be safer than existing vehicles, newly licensed drivers are supposed to be better educated than drivers in the past. Moreover, society creates and changes rules for traffic behaviour in order to improve the safety of the system. These renewal and growth processes of roads, vehicles, drivers and rules in the traffic system result in an adaptation of the system to a steadily safer system. In this view growth and renewal are inherently related to the safety of the system. Without growth and renewal there is hardly any enhancement of safety conceivable.

Growth of vehicle kilometers is not unlimited. The number of actual drivers is restricted by the number of the population and by time available for travelling. The main limitation, however, is the available length of road-lanes. This is not only restricted by economic factors, but has a limit by the limits of space, especially in densely populated areas. We conjecture therefore a still unknown saturation level for the number of vehicle kilometers, viz. a limit for growth of traffic. An interesting question we try to answer is, to which extent such a limit of growth also imposes, by its postulated inherence for safety, a limit to the attainable level of safety.

3. MATHEMATICAL DESCRIPTION OF GROWTH

3.1. Absolute growth

From inspection of the curves for vehicle kilometers over a long period in many countries, it can be deduced that these growth curves in the starting phase are of an exponentional increasing nature. Fore some countries a decreasing growth seems apparent in the more recent periods, however not always evidently different from a somewhat irregular linear increase. On the other hand the theoretical notion of some unknown future saturation level or at least a notion of limits of growth for vehicle kilometers has strong face-validity. On the basis of these considerations we restrict ourselves to growth described by sigmoid curves. We will concentrate on three types of sigmoid curves with time as the independent variable often used in sociometrics and econometrics, leaving other types used in ecology (May & Oster, 1976) aside. In the literature (Mertens, 1973; Johnston, 1963; Day, 1966) on econometrics and biometrics, these sigmoid growth curves are well documented. These three growth curves are named as the logistic curve based originally on the well-known Verhulst equation (Verhulst, 1844), the Gompertz curve originated by Gompertz (1825) and the log-reciprocal curve traditionally used in econometrics (Prais & Houthakker, 1955; Johnston, 1963).

Let: V_t =vehicle kilometers in year t

V_{max}=saturation level veh. km. for $t \to \infty$

α, β =parameters

t =time in years

we write these curves as comparable exponentional functions

logistic curve

$$V_t = V_{max} \left[1 + e^{-(\alpha t+\beta)} \right]^{-1} \qquad (\alpha > 0) \qquad (1)$$

Gompertz curve

$$V_t = V_{max}\, e^{-e^{\alpha t+\beta}} \qquad (\alpha > 0) \qquad (2)$$

log-reciprocal curve

$$V_t = V_{max}\, e^{-(\alpha t+\beta)^{-1}} \qquad (\alpha > 0) \qquad (3)$$

In Figure 6 we give an impression of the shape of these curves

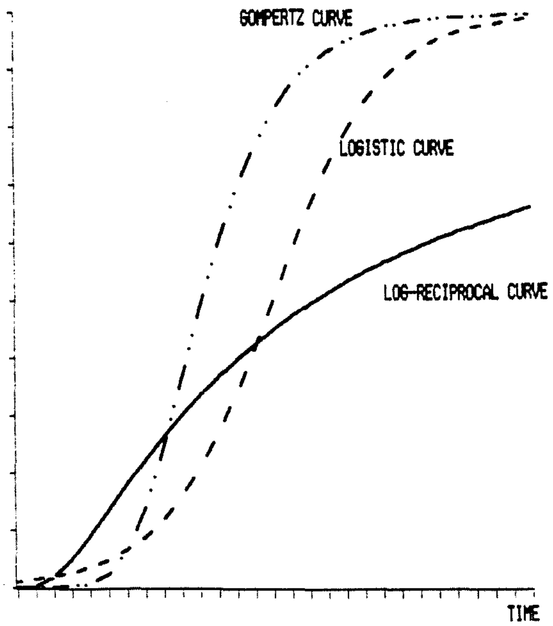

Figure 6. Curves of growth with saturation.

If we take V_t/V_{max} as the proportion of growth realized in year t, we see that these curves due to the exponentional expressions range from zero to unity with time progressing.
Since it is not so much vehicle kilometers that saturate, but density of traffic as a demand-supply relation between length of road lanes and distances travelled, a transformation from vehicle kilometers to density may be in place. Enlargement of length or road lanes in our system approach is a lagged reaction on the growth of vehicle kilometers. A transformation by a monotonic continuous reducing function of the vehicle kilometers themselves, therefore, may be an appropriate transformation. Such a transformation leads to a generalization of functions for growth.

. As from the theory of traffic flow (Haight, 1963), it is well known
. that the mean of the distribution of vehicles on the lanes (Poisson
. distribution) is directly related to the mean of the density
. distribution (negative exponentional distribution), a power-
. transformation as a monotonic continuous transformation of vehicle
. kilometers itself has theoretical justification.
. Assuming that the development of mean density of traffic over time,
. defined as D_t , can be expressed by a power-transformation of vehicle
. kilometers
.

$$D_t = u\, V_t^c \qquad (c < 1) \qquad (4a)$$

If the reduction is due to the lagged enlargement we may even conjecture that density is dependent on a lagged value of V_t. If the time-lag is τ and $t - \tau = t'$ (4a) becomes

$$D_t = u\, V_{t'}^{c} \qquad (c < 1) \qquad (4b)$$

This last expression will also be valid if we include the dependence on actual vehicle kilometers by a weighted geometric mean of (4a) and (4b), since this mean is fairly exact represented by a time-lag between τ and 0.

By reciprocal powering both sides of the curve-equations we see that this power-transformation of (4a) is absorbed in the β-parameter of the Gompertz curve and in the α and β parameters of the log-reciprocal curve. The logistic curve becomes asymmetric (Nelder, 1961) and for reasons of comparability of notation this generalization is written as

<u>asymmetric logistic curve</u>

$$V_t = V_{max} \left[1 + e^{-(\alpha t+\beta)} \right]^{-1/c} \qquad (5)$$

For $c < 1$ the logistic curve moves toward the Gompertz curve and for $c > 1$ this curve is described by a slower increase in the beginning and a quicker levelling off at the end.

An other generalization is obtained by a similar monotonic transformation of the time axes.

Since scale and origin of time are undetermined this power-transformation replaces time as $\alpha t+\beta$ in the equations by $(\alpha t+\beta)^k$. Except for the log-reciprocal curve we shall not elaborate on this last generalization, because of the rather complex nature of its derivatives on which we concentrate hereafter. This generalization of the log-reciprocal curve is written as

<u>generalized log-reciprocal curve</u>

$$V_t = V_{max}\, e^{-(\alpha t+\beta)^{-k}} \qquad (k > 0) \qquad (6)$$

3.2. Increase of growth

The increase of growth is mathematically described by the derivative of the functions for growth.

Writing the derivatives of (2), (5) and (6) with respect to time, we obtain the functions of time for the increase of vehicle kilometers corresponding to the growth models given above as

derivative of asymmetric logistic curve

$$V_t^* = \alpha\, c^{-1}\, V_{max}\, [\, 1 + e^{-(\alpha t+\beta)}\,]^{-1/c}\, [\, 1 + e^{\alpha t+\beta}\,]^{-1} \qquad (7a)$$

or

$$V_t^* = \alpha\, c^{-1}\, V_t\, [\, 1 + e^{\alpha t+\beta}\,]^{-1} \qquad (7b)$$

or after some further substitution and manipulation of (5)

$$V_t^* = \alpha\, c^{-1}\, V_{max}^{-c}\, V_t\, [\, V_{max}^{c} - V_t^{c}\,] \qquad (7c)$$

derivative of Gompertz curve

$$V_t^* = \alpha\, V_{max}\, e^{-e^{\alpha t+\beta}}\, e^{\alpha t+\beta} \qquad (8a)$$

or

$$V_t^* = \alpha\, V_t\, e^{\alpha t+\beta} \qquad (8b)$$

or after some further substitution and manipulation of (2)

$$V_t^* = \alpha\, V_t\, [\, \ln V_{max} - \ln V_t\,] \qquad (8c)$$

derivative of generalized log-reciprocal curve

$$V_t^* = \alpha^k\, k^{-1}\, V_{max}\, e^{-(\alpha t+\beta)^{-k}}\, (\alpha t+\beta)^{-(k+1)} \qquad (9a)$$

or

$$V_t^* = \alpha^k\, k^{-1}\, V_t\, (\alpha t+\beta)^{-(k+1)} \qquad (9b)$$

or again after some further substitution and manipulation of (6)

$$V_t^* = \alpha^k\, k^{-1}\, V_t\, [\, \ln V_{max} - \ln V_t\,]^{(k+1)/k} \qquad (9c)$$

From (8c) and (9c) we see that for $k \to \infty$, the Gompertz curve is a limit case of the generalized log-reciprocal curve, since $\lim_{k \to \infty} (k+1)/k = 1$ and α can be redefined.

The relation between the asymmetric logistic curve and the Gompertz curve can also be described as a limit case for $c \to 0$. Rewriting (7c) as

$$V_t^* = \alpha V_t \left[\frac{1 - (V_t/V_{max})^c}{c} \right]$$

and noting that

$$\lim_{\tau \to 0} \frac{X_t^\tau - 1}{\tau} = \ln X_t \qquad (10)$$

we find for $c \to 0$ by substituting $\tau = c$ and $X_t = V_t/V_{max}$

$$\lim_{c \to 0} V_t^* = \alpha V_t [\ln V_{max} - \ln V_t]$$

Since this is identical to (8c) we see that, for the power-transformation parameter c approaching to zero, the Gompertz curve is also the limit case for the asymmetric logistic curve.

From a more phenomenal level it is also interesting to calculate the inflexion point of these curves, because inflexion points determine the maximum increase in vehicle kilometres with respect to time.

By setting the second derivative with respect to time to zero and substituting these time values into (1) or (5), (2) and (3) or (6), the maximum increments for these curves are obtained. For c=1 in (7) this gives $t = -\beta/\alpha$ or at the time where

$$V_t = 0.5\, V_{max} \qquad \text{(logistic curve)}$$

and for any $c > 0$ in (7) at the time where

$$V_t = (c+1)^{-1/c}\, V_{max} \qquad \text{(asymmetric logistic curve)}$$

Note that $(c+1)^{-1/c}$ for $c = 1$ becomes 0.5 and for $\lim c \to 0$ this term approaches $e^{-1} = 0.3678$, because of the well-known definition of e as the limit of $(1+1/n)^n$ for $n \to \infty$.

. We also obtain for (8) $t=-\beta/\alpha$ or at the time where

$$V_t = 0.3678\ V_{max} \qquad \text{(Gompertz curve)}$$

and for k=1 in (9) we obtain $t=(1-2\beta)/2\alpha$ or at the time where

$$V_t = 0.135\ V_{max} \qquad \text{(log-reciprocal curve)}$$

and for any $k > 0$ at

$$V_t = 0.3678^{(k+1)/k}\ V_{max} \qquad \text{(generalized log-reciprocal curve)}$$

Again note that $0.3678^{(k+1)/k} = 0.135$ for $k = 1$.

In Figure 7 we picture the development of the increase in vehicle kilometers as derivatives of the standard non-generalized curves in correspondence to Figure 6.

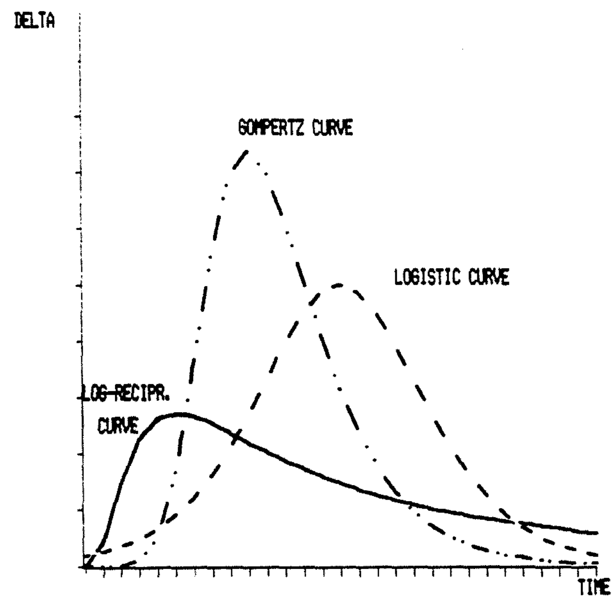

Figure 7. Curves of the increase of growth.

In Figures 6 and 7 and the computations just given, the inflexion point of the sigmoid curves comes earlier for the log-reciprocal curve than for the Gompertz curve and the inflexion point for the logistic curve is situated later than for the Gompertz curve.

By the power-transformation, with c going from unity to zero, the vertical axes can be compressed so that the asymmetric logistic curve approaches the form of the Gompertz curve from one side. From the other side the form of the Gompertz curve is approached by the power-transformation of the horizontal time-axes, with k going from unity to infinity, for the log-reciprocal curve. This transformation stretches and compresses time around the point where rescaled time is unity (viz. $t = (1-\beta)/\alpha$).

The asymmetric logistic curve and the generalized log-reciprocal curve therefore seems to span the space of possible sigmoid curves fairly well. In general, the log-reciprocal curve takes longer to level off than the logistic curve. These considerations may also guide the choice of type of curve on a phenomenal level.

3.3. Acceleration of growth

As shown by (7c), (8c) and (9c) all these sigmoid shaped curves are described by an increase of growth as the product of the growth achieved and (a transformation of) the growth still possible. This property leads to a very interesting aspect related to the mathematical description of adaptation since it enables one to write the rate of increase of the growth curve, defined as acceleration, by relatively simple functions which turn out to be monotonically decreasing functions of time.

Let: $$Q_t = V_t^*/V_t \qquad (11)$$

We write from (7b), (8b) and (9b) the different accelerations Q_t as

asymmetric logistic acceleration

$$Q_t = \alpha\, c^{-1} \left[1 + e^{\alpha t+\beta} \right]^{-1} \qquad (12)$$

We see from (12) that the shape of the acceleration curve for asymmetric logistic growth is not effected by the generalizing power-transformation of V_t and remains symmetric.

Gompertz acceleration

$$Q_t = \alpha\, e^{\alpha t+\beta} \qquad (13)$$

- log-reciprocal acceleration
-
- $$Q_t = \alpha^k k^{-1} [\alpha t+\beta]^{-(k+1)} \qquad (14)$$
-
- Thus, the generalizing power-transformations on time for log-
- recoprocal growth is with respect to the acceleration equivalent to a
- power-transformation of the acceleration itself.

In Figure 8 we show these acceleration curves (for c=1 and k=1).

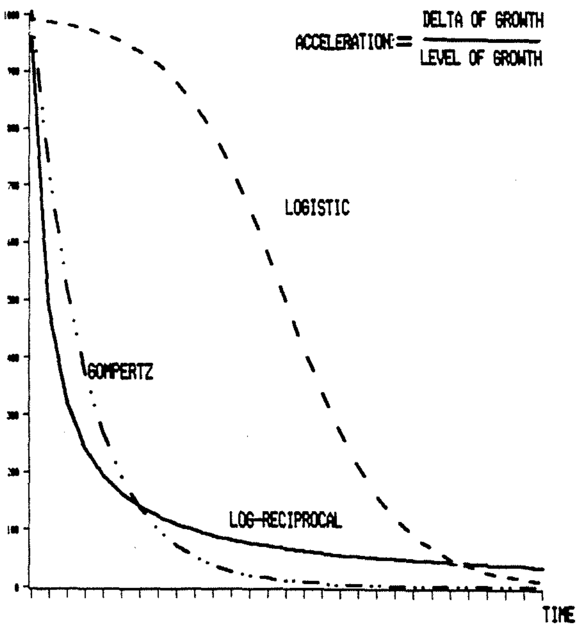

Figure 8. Curves for the acceleration of growth.

As can be seen from the formulae and the graphs these acceleration curves are monotonicly decreasing curves and as such can be candidates for a mathematical description of adaptation in time.

3.4. Growth and probability functions

These explicit formulae for growth with saturation are commonly found in the literature, but are by no means exhaustive. Referring to the proportion of growth as values between zero and unity and considering the graphs in Figure 7, we may think of continuous single-peaked density functions of distributions in probability theory for which every cumulative probability function forms a legitimate sigmoid-shaped description for the proportion of growth.

- Unfortunately most cumulative probability-distribution functions do
- not have explicit formulae and therefore cannot be treated as
- described. However, integration of such probability distributions
- for which no explicit formulae exist, is approximated by summation of
- small discrete steps.

Such sum functions describe growth of traffic volume as the achieved proportion of a saturation level by sigmoid curves. The many probability distributions that fall into this class, show that the number of possible and mathematical tractable growth curves are not limited to those mentioned here or otherwise found in the literature.

4. MATHEMATICAL DESCRIPTION OF ADAPTATION

4.1. Interpretations of risk reduction

The decreasing fatality rate has been interpreted by Koornstra (1987) and Minter (1987) as a community learning process.
Their interpretations, however, differ. Minter stresses collective individual learning, where Koornstra points to a gradual learning process of society by enhancing safety through changes in road network, vehicles, rules and individual behaviour. Minter's interpretation is in accordance with stochastic learning theory (Sternberg, 1967), where learning is a function of the number of events. Koornstra's interpretation leads to community learning as a function of time. This last interpretation could be named "adaptation", since generally adaptation is a function of time.

Koornstra (in Oppe et al., 1988) rejects Minter's interpretation on two grounds. In the first place the fatality rate decreases more than the injury rate, which in Minter's interpretation means that individuals learn to discriminate and avoid fatal-accident situations better than less severe accident situations. This cannot be explained by individual cumulative experience. Secondly the mathematical learning curve functions described by Koornstra and Minter do fit the data much better as a function of time, than as a function of the cumulative experience, expressed by the sum of vehicle kilometers as Minter does.

On the other hand, transforming mathematical learning theory as functions of the number of relevant events (trials) to functions of time asks for strong assumptions. These assumptions are contained in our "closed" system interpretation of traffic and the adaptation theory of Helson (1964). The concept of adaptation as time-related adjustment to environmental conditions, must be brought in accordance to the event-related improvement described in learning theory.
Our "closed" self-organizing system interpretation points to the gradually safer conditions, while growth of traffic as such leads to more accidents. Growth of traffic, however, also implies safer renewal, enlargement of a safer road network, safer vehicles and better and coordinated rules. These effects are not immediate but generally will lag in time. New laws, like belt laws, lead to belt-wearing percentages gradually growing in time. Reconstructions of black-spots are reactions

of communities on a growing number of accidents leading to a reduction of accidents later. Traffic growth leads to building motorways, which after long periods of building-time attract traffic to these much safer roads.

In our view counter-effects may only partially occur by risk compensation (Wilde, 1982), such as present in gradually rising speeds of road traffic. These rising speeds are made possible by better roads and cars, but the cars are not only constructed for higher speeds; they are also inherently safer by crash zones, soft interior materials, better or semi-automatic breaking mechanism and so on. Helson's adaptation theory states that behavioural adaptation is the pooled effect of classes of stimuli, such as focal, contextual and internal stimuli. The level of adaptation is a geometric weighted mean of all kinds of stimuli. Helson's theory of adaptation level is different from homeostasis theory, as expressed by Wilde (1982), "because it stresses <u>changing levels</u>" (quotations from page 52, Helson, 1964). The fact that adaptation level is a weighted mean of different classes of stimuli implies that influence of one class may be counteracted by other classes of stimuli, but also that the influence of one class of stimuli may dominate over other classes of stimuli. Since in the period of emergence of motorized traffic the nature of man did not change so much, while the physical and social environment has changed dramatically, the apparent drop in risk as the change in level of adaptation must be contributed mainly to the inherently safer external conditions.

Taking into account the graduality of change in traffic environment, the lagged and over many years integrated safety effects and the eventually partial and lagged counter-activity of human behaviour, we conjecture that adaptation to safer traffic is better described by a function of time, than as a function of cumulative traffic volume.

4.2. <u>Learning theory and adaptation</u>

Referring to the incorporation of Helson's theory in the theory of social and learning systems (Hanken & Reuver, 1977) one possibility is to assume that the adaptation process reduces the probability of a fatal accident under equal exposure conditions by a constant factor per time-interval.

Hence

$$P_{t+1} = \delta \, P_t \tag{15}$$

Comparing this equation with mathematical learning theory, we assume a model similar to Bush and Mosteller (1955) in their linear-operator learning theory or to the generalized and aggregated stimulus-sampling learning theory of Atkinson and Estes (Sternberg, 1967; Atkinson & Estes, 1967). The difference is that now time is the function variable, instead of n, the number of (passed) relevant learning events, since in the Bush-Mosteller or linear-operator learning model the probability of error is reduced by a constant factor at any learning event.

Recursive application of (15) gives

$$P_{t+1} = \delta^t P_1$$

Denoting $P_1 = e^b$ and $\delta = e^a$, we arrive at the basic expression of the <u>linear-operator model</u>

$$P_t = e^{at+b} \qquad (a < 0) \qquad (16)$$

Sternberg (1967) compared the existing learning models and summarized that generally these models are based on a set of axioms, characterized by

- path independence of events
- commutativity of effects of events
- independence of irrelevant alternatives or arbitrariness of definition of classes of outcomes of events

while aggregation over individuals (mean learning curves) also postulates:

- valid approximation of mean-values of parameters or scales assuming distributions over individuals concentrated at its mean.

On these assumption two other learning models have been developed, the so-called beta-model from Luce (1960) and the so-called urn-model from Audley & Jonckheere (1956). The urn-model has its roots in the earliest mathematical learning models of Thurstone (1930) and Gulliksen (1934). In the same way as for the linear-operator model these models can be reformulated as time-dependent adaptation models.

Luce assumes the existence of a response-strength scale v, in the tradition of Hullian learning theory (Hull, 1943), for a particular type of reaction. The error probability at the n+1 event is reduced by a reduction of the response strength for that error by a factor β ($\beta < 1$) in such a manner that

$$P_{n+1} = \frac{\beta\, v_n}{1 + \beta\, v_n} \qquad \text{and} \qquad v_n = \frac{P_n}{1 - P_n}$$

From which it follows that

$$P_{n+1} = \frac{\beta\, P_n}{(1 - P_n) + \beta\, P_n}$$

Similar aggregation over response classes and individuals as for the linear-operator model by Helson's adaptation-level theory, allows us to assume an aggregate safety scale v_t for the community that changes according to our social self-organizing system description by a factor β with time and arrive at

$$P_{t+1} = \frac{\beta\, v_t}{1 + \beta\, v_t} \qquad (17)$$

Recursive application of (17) leads to

$$P_t = [\ 1 + \beta^{-t} v_1\ ]^{-1}$$

Substituting $v_1 = e^b$ and $\beta = e^{-a}$ we obtain the basic expression as <u>beta-model</u>

$$P_t = [\ 1 + e^{at+b}\ ]^{-1} \qquad (a < 0) \qquad (18)$$

The urn-model in its earliest description by Thurstone assumes that the reciprocal of the error probability increases with an additive constant a per learning event, such that

$$P_{n+1}^{-1} = P_n^{-1} + a$$

One of the many possible reformulations of the urn-model as described by Audley & Jonckheere (1956), in the spirit of our renewal and growth process of traffic, could be as follows.
The probability of a fatal accident in time interval t, is proportional to the ratio of situations liable to fatal accidents (r_t) and the sum of situations liable to fatal accidents and all

other safer situations ($r_t + w_t$) (red and white balls in the urn). Through self-organizing the number of safer situations is enlarged with c situations in time interval t. Assuming that self-organization by growth (adding safe and dangerous situations) and renewal (partially turning dangerous situations into safe ones) leaves the number of situations liable to fatal accidents unchanged, we obtain

$$P_{t+1} = \frac{r_t}{r_t + w_t + c}$$

Recursive application leads to

$$P_{t+1} = \frac{r_1}{r_1 + w_1 + c\,t}$$

Denoting $b = (r_1+w_1)/r_1$ and $a = c/r_1$ we arrive at the basic expression for the
<u>urn-model</u>

$$P_t = [\ at + b\ ]^{-1} \qquad (a > 0) \qquad (19)$$

which is equivalent to the Thurstonian model with t instead of n.

In Figure 9 we demonstrate the behaviour of these adaptation models.

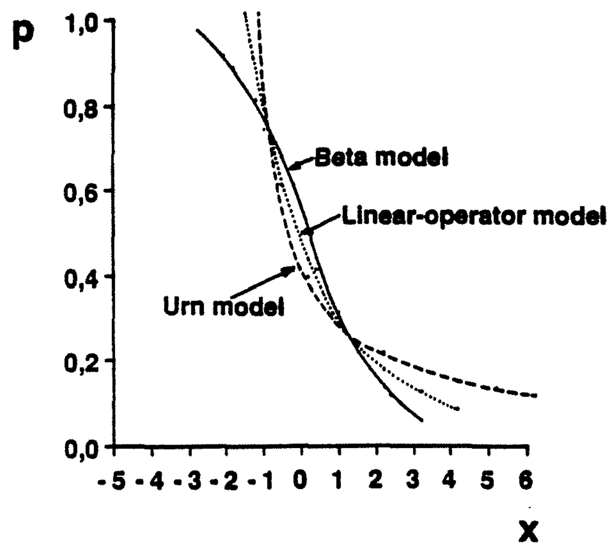

Figure 9. Nomogram for models of adaptation.

It will be noted that time has no origin nor a unit of scale. Therefore linear transformation of time (generally with positive small scaling factor and large negative location displacement if t is taken in years A.D.) are permissible and do not change the general algebraic expressions for the functions of adaptation with time. Taking the parameters of the time axes, denoted by X, in such a manner that P_t=0.25 and P_t=0.75 coincide for the three models in Figure 9 (monogram taken from Sternberg, 1967, p.51), we are able to inspect the different behaviours of the models more closely.

4.3. Generalization of adaptation models

Just like the growth curves of the growth-models we may generalize our adaptation expressions by a similar transformation.

. A power-transformation of P_t is equivalent to a monotonic time-
. dependent transformation of the reduction factor of the decrease in
. P_{t+1} with respect to P_t. Thereby we replace the axiom of path
. independence by a semi-independence axiom, which is appropriate to
. our time related functions.
. Since this transformation is absorbed in the parameters of (16) the
. linear-operator model remains unchanged; but in (19) and (20) the
. power of -1 for the beta-model and urn-model has to be replaced by a
. negative parameter.

According to these mathematical descriptions, the probability of a fatal accident will reduce to zero with time progressing infinitely.
Along the lines of Bush and Mosteller (1955) we may also introduce imperfect adaptation to a non-zero level as another generalization.

. This results in multiplication with $(1-\pi)$ and addition of π for our
. model expressions.
. Rewriting the adaptation models for these two generalizations, we
. obtain
.
. generalized beta-model
.

$$P_t = (1-\pi)\ [\ 1 + e^{at+b}\ ]^{-1/r} + \pi \qquad (21)$$

.

. <u>generalized linear-operator model</u>

$$P_t = (1-\pi)\ e^{at+b} + \pi \tag{22}$$

. <u>generalized urn-model</u>

$$P_t = (1-\pi)\ [\ at + b\ ]^{-1/m} + \pi \tag{23}$$

Koornstra (1987), Oppe (1987) and Haight (1988) used the linear-operator model for the fit of the fatality rate on the assumption of reduction to zero and of fatality rate as the probability of fatalities (P_t). They found a remarkable good fit for the data of time-series for the USA, Japan, FRG, The Netherlands, France and Great-Britain over periods ranging from 26 to 53 years.
Minter (1987) used Towill's learning model (Towill, 1973), which as Koornstra (in Oppe et al., 1988) proved, is essentially the beta-model under the condition that time as the independent variable is replaced by the cumulative sum of vehicle kilometers as an estimation of the collective number of past learning events.

The fatality ratio is defined as a probability. It is, however, by no means assured that the fatality rate is a probability measure. In order to be a probability the number of fatalities should not be related to traffic volume but to exposure as the expected number of possible encounters liable to fatalities.
Among others Koornstra (1973) and Smeed (1974) argued that exposure is quadraticly related to the density. The strict arguments for a quadratic relation are based on independence of vehicle movements. On theoretical grounds increasing dependence of vehicle movements in denser traffic is conjectured by Roszbach (in Oppe et al., 1988), stating that exposure will grow slower with increasing vehicle kilometers than assumed on growth of density without queue's and platoons. Since dependence increases with increasing density we assume that dependence reduces growth of exposure by a power-transformation of the squared density itself.

. Referring to (4) it follows that exposure also develops in time
. according to power-transformation of V_t.
. Denoting exposure at time interval t as E_t, we write

$$E_t = g\, D_t^{2z} = d\, V_t^s \qquad (z < 1) \qquad (24a)$$

With reference to (4) we see that $2\,c\,z = s$, while $c < 1$ and $z < 1$, so whether V_t as vehicle kilometers is a fair approximation of exposure as E_t depends on the approximation of s to unity. From the assumptions made it is deduced that $0 < s < 2$ and that s will be the smaller the denser traffic is.
If we assume as in (4b) that growth of density is lagged with respect to growth of vehicle kilometers, the alternative expression becomes

$$E_t = g\, D_t^{2z} = d\, V_{t'}^s \qquad (z < 1) \qquad (24b)$$

Now the probability of a fatality legitimately can be written as the ratio of the number of fatalities and exposure.

This is written as

$$P_t = \frac{F_t}{d\, V_t^s} \qquad (25a)$$

where d and s are parameters according to (24a) or in accordance with (24b) written as

$$P_t = \frac{F_t}{d\, V_{t'}^s} \qquad (25b)$$

By taking this ratio of fatalities and exposure as the probability measure for the adaptation models we complete the mathematical description of adaptation.

5. RELATIONS BETWEEN GROWTH AND ADAPTATION

5.1. General mathematical relations

Instead of analyzing and fitting curves to observed data for the different models of growth and of adaptation separately, we concentrate on the conceptually postulated intimate relation between growth and adaptation. In the spirit of our system-theoretical approach we directly express mathematical relations between acceleration and adaptation. We demonstrate that such a relation can be established in a fairly general way, more or less independent from the particular growth model or adaptation model. We regard the generality of this relation between adaptation and growth as the basic result from our theory.
In the paragraph on the mathematical description of growth curves we stated that the expressions for acceleration curves are monotonically decreasing curves and as such are candidates for the description of adaptation. Indeed, if we compare on a phenomenal level Figure 8 with graphs of the three models of growth and Figure 9 with the three adaptation curves we see, apart from differences in location and scale of time, identical shapes of curves for

logistic acceleration	≈	beta-model adaptation
Gompertz acceleration	≈	linear-operator model adaptation
log-reciprocal acceleration	≈	urn-model adaptation

Comparing the expressions for acceleration with the expressions for adaptation, we see a one to one correspondence (if $\pi = 0$ assuming zero fatalities at the end of the process) between the above-mentioned pairs of curve expressions. This mathematical correspondence enables one to express adaptation as mathematical function of acceleration, which is in fact based on the same relation as in the ecological system between the number of mature survivors and immature non-survivors pictured in Figure 2.
The task is to relate time in the growth process (expression $(\alpha t+\beta)$ of Q_t) in a meaningful way to time in the adaptation process (expression $(at+b)$ of P_t). Since both expressions are linear functions of time with two parameters we need two other parameters to relate these expressions linearly without constraints. Because of the linear nature these two parameters are one parameter for difference of location of time and one parameter for ratio of scales in time.

The difference of location of time can be interpreted as a time-lag between the growth process and the adaptation process. In our closed-system description growth precedes adaptation, hence a time-lag of τ in units of t for the time-scale of adaptation with respect to t` the time-scale of the growth process. The ratio of units of time-scales, defined by q, will be unity if the processes develop with the same speed in time. This seems most likely, but is not a necessary assumption. If q should be unequal to unity either growth or adaptation is a faster process. Within the closed adaptive self-organizing system interpretation, however, we are inclined to think of adaptation as a lagged process at approximately equal speeds, compared to the growth process.

Formally writing t` for time in the growth expressions, we obtain

$$at + b = (\alpha t' + \beta)\ q \qquad (26a)$$

$$t = t' + \tau \qquad (26b)$$

stating that the relation between parameters is given by

$$a = \alpha q \qquad (27a)$$

$$b = \beta q + \alpha \tau q \qquad (27b)$$

We conjecture on the basis of the above given interpretation as a closed adaptive system that

$$q \approx 1 \qquad (28a)$$

$$\tau \geq 0 \qquad (28b)$$

Substituting (26a) in (12), (13) and (14) we write these expressions in t` and t, assuming τ to be known or to be estimated independently. Relating these expressions to (21), (22) and (23) we obtain between $Q_{t'}$ and P_t equations which only depend on q and some free parameters, but are no longer dependent on a and b or α and β, since e^{at+b} as function of P_t is substituted in $e^{(\alpha t' + \beta) q}$ as function of $Q_{t'}$.
For one part we write according to the generalized beta-model of (21) for π=0

$$e^{at+b} = P_t^{-r} - 1 \qquad (29)$$

and to the generalized linear-operator model of (22) for π=0

$$e^{at+b} = P_t \qquad (30)$$

and lastly to the generalized urn-model of (23) for π=0

$$e^{at+b} = e^{P_t^{-m}} \qquad (31)$$

For another part we write the acceleration curves in t' for the three growth models with the exponentional term on one side. This results for the asymmetric logistic acceleration of (12) in

$$e^{\alpha t' + \beta} = \sigma\, Q_{t'}^{-1} - 1 \qquad \sigma = (a/c)^{-1} \qquad (32)$$

for the Gompertz acceleration of (13) in

$$e^{\alpha t' + \beta} = \sigma\, Q_{t'} \qquad \sigma = \alpha^{-1} \qquad (33)$$

and at last for the log-reciprocal acceleration of (14) in

$$e^{\alpha t' + \beta} = e^{\sigma\, Q_{t'}^{-1/(k+1)}} \qquad \sigma = \alpha^{-k}\, k \qquad (34)$$

By substituting (26a) into (32), (33) and (34) we obtain nine relatively simple equivalence relations for all pair-wise combinations with (29), (30) and (31). This is summarized in Table 1 as direct expressions of P_t into $Q_{t'}$.

e^{at+b} equals $e^{(\alpha t'+\beta)q}$	ADAPTATION		
	gen. beta-model $P_t^{-r} - 1$	lin.-oper. model P_t	gen. urn-model $e^{P_t^{-m}}$
ACCELERATION: logistic $\{\sigma Q_{t'}^{-1} - 1\}^q$	$P_t^{-r} - 1 = \{\sigma Q_{t'}^{-1} - 1\}^q$	$P_t = \{\sigma Q_{t'}^{-1} - 1\}^q$	$P_t^{-m} = q[\ln\{\sigma Q_{t'}^{-1} - 1\}]$
ACCELERATION: Gompertz $\{\sigma Q_{t'}\}^q$	$P_t^{-r} - 1 = \{\sigma Q_{t'}\}^q$	$P_t = \{\sigma Q_{t'}\}^q$	$P_t^{-m} = q[\ln\{\sigma Q_{t'}\}]$
ACCELERATION: log-recipr. $e^{\sigma Q_{t'}^{-q/(k+1)}}$	$\ln\{P_t^{-r} - 1\} = \sigma Q_{t'}^{-q/(k+1)}$	$\ln\{P_t\} = \sigma Q_{t'}^{-q/(k+1)}$	$P_t^{-m} = \sigma Q_{t'}^{-q/(k+1)}$

Table 1. Relations between adaptation curves and acceleration curves (in the last row σ is redefined as $\sigma^{-q/(k+1)}$)

. With help of the well-known generalized power-transformation,

$$g(X_t) = X_t^{(\tau)} = \begin{cases} \ln X_t & \tau=0 \\ (X_t^{\tau}-1)/\tau & \tau\neq 0 \end{cases}$$

. based on the limit given in (10), and often used in transformations (see Box & Cox, 1964; Krzanowski, 1988) for normality of distribution, for additivity or homogeneity of variance, in all cases of Table 1 the relation between $Q_{t'}$ and P_t can be written as

. <u>general assumption</u>

$$g_0\{ g_1\{ P_t \}\} = g_2\{ x \{ g_3\{ Q_{t'} \} + w \}^z + y \qquad (35)$$

. Hereby the respective logistic, logarithmic or power-transformations in Table 1 of P_t and/or $Q_{t'}$ can be generated.

The general conclusion therefore is that the curves of acceleration for all models of saturating growth for positive outcomes are monotonically related to the curves of adaptation models for negative outcomes in the same system.

5.2. <u>Simplifications</u>

From our closed self-organizing adaptive system-interpretation we conjecture corresponding processes for growth and adaptation. This implies not only corresponding model descriptions, but at least also equal speeds of processes (viz. q=1).

. For the pair-wise relations of the diagonal of Table 1 it follows that the relation simplifies to our

. <u>basic assumption</u>

$$P_t = \delta\, Q_{t'}^{\mu} \qquad (37a)$$

. where δ and μ are free parameters. Referring to Table 1 we see that (for q=1), if the generalization parameters for power-transformation of $Q_{t'}$ or P_t are process-related (viz. r=1, m=1/[k+1]), μ=1; stating that in these cases P_t is even proportional to $Q_{t'}$.

Based on correspondence between models for growth and adaptation, this plausible simplification leads to the basic assumption of our theory, which states that the monotonic relation between acceleration and adaptation is a proportional power-function.

Substituting (25) and (11), the definitions of P_t and $Q_{t'}$, into (37a) the expression becomes either by (25a)

$$\frac{F_t}{d\,V_t^s} = \delta \left[\frac{V_{t'}^*}{V_{t'}} \right]^{\mu} \qquad (37b)$$

or by (25b) for equal time-lags in (37a) and (24b)

$$\frac{F_t}{d\,V_{t'}^s} = \delta \left[\frac{V_{t'}^*}{V_{t'}} \right]^{\mu} \qquad (37c)$$

Since d and δ are both free proportionality parameters we can set d=1 without loss of generality.

Further simplifications are possible by some approximations.

Noting that if

a) - either $t \approx t'$ (violating the time-lag assumption)

- or Q_t is proportional to $Q_{t'}$ (which is exactly true for the Gompertz acceleration)
- or P_t is proportional to $P_{t'}$ (which is exactly true for the linear-operator model)
- or V_t is proportional to $V_{t'}$ (which is exactly true for exponentional growth of vehicle kilometers, but violates our saturation assumption)
- or approximate proportionality applies to or is well approximated by corresponding departures of proportionality for P_t and Q_t

we can by redefining δ, as including the proportionality-factor, replace t` by t in (37b) as a simplification of our basic assumption; or if

b) - either proportionality for V_t holds approximately (again violating the saturation assumption)

- or the time-lag assumption of (4b) holds and the equivalence of time-lags for (37c) is well approximated,

we obtain by (37c) itself or by replacing t by t` for V_t in (37b) another simplified form of our basic assumption simplified basic assumption

in case a) $F_t \approx \delta \, [\, V_t^* \,]^{\mu} \, V_t^{s-\mu}$ (38a)

in case b) $F_t \approx \delta \, [\, V_{t'}^* \,]^{\mu} \, V_{t'}^{s-\mu}$ (38b)

If $s = \mu$ this simplifies to our

specific assumption

in case a) $F_t \approx \delta \, [\, V_t^* \,]^{\mu}$ (39a)

in case b) $F_t \approx \delta \, [\, V_{t'}^* \,]^{\mu}$ (39b)

If also $s = 1$, thereby assuming that exposure is well approximated by vehicle kilometers, it follows that $\mu = 1$. Thereby equivalence of process speeds ($q=1$) and related generalization parameters ($q/r=1$ or $q/(k+1)=m$ for growth and adaptation as well as the validity of some condition in case a) or b) is assumed. The ultimate simplification under these restrictive assumptions becomes the

simplified specific assumption

in case a) $F_t \approx \delta \, V_t^*$ (40a)

in case b) $F_t \approx \delta \, V_{t'}^*$ (40b)

These last simplifications result in a proportional (power-)relation between fatalities and the increase in vehicle kilometers. Although all these restrictions may seem to be based on rather strong assumptions, the data analyses for several countries by Oppe (1987) and by Koornstra (in Oppe et al., 1988) support such ultimately simple relations.
This suggests at least that

- growth and adaptation can be conceived as closely related and that the mathematical theory has validity
- some strong simplifications in the theory are adequate
- the transformations to density and exposure is such that exposure is well approximated by vehicle kilometers.

5.3. Generalized simplification

Although we generated no other adaptation models than the ones corresponding to (a generalization of) the well-known learning models, we could refer to our extension of growth models as functions for the proportion of growth taken from cumulative probability functions. In line with such an extension we conjecture that legitimate adaptation models, having all the referred properties of the learning models as discussed by Sternberg (1967), are formed by any function described by a single-peaked probability-density function divided by its cumulative function.

Enlarging the set of growth models and that of adaptation models in that way it is tempting to assume that for any growth model, there always exists an adaptation process in such a manner that the basic assumption holds.

Our generalized basic assumption, irrespective of the type of growth model or adaptation model, for any self-organizing system characterized by growth of positive outcomes and adaptation to (a near zero level) negative outcomes, can be formulated as follows:
The probability of a negative outcome is proportional to a power-transformation of the acceleration of the growth process of positive outcome in any self-organizing system.

Positive and negative outcomes in this system description are related and cannot be defined arbitrarily. Negative outcomes therefore must be defined as self-defeating events for positive outcomes. In biology this may be premature non-survival defeating growth of population; in the traffic system it may be events (fatal accidents) that wash out mobility.

6. EMPIRICAL EVIDENCE

6.1. Calculations and approximations

The validity of the basic assumption can be investigated by the analyses of data from several countries. We do this by graphical presentations of data on fatalities and fatality rates, and after some simple calculations and approximations from the data, also on growth and acceleration of vehicle kilometers. This is possible without curve fitting for growth or adaptation separately, since the main elements of (37b, 37c) can be constructed from or consist of observable values.

The right-hand side of (37) contains the increase of growth as the derivative of vehicle kilometers. Without fitting a particular growth model to vehicle kilometers, this asks for the calculation of an approximation of the value of the increase directly from the data. We choose to compute the increase by approximate values for the derivative through finite difference calculus.
However, computing the increase from differences in vehicle kilometers per year may lead to negative estimates due to (economic) fluctuations of vehicle kilometers from year to year, whereas the number of fatalities is always positive. Moreover, in our theory fatalities are outcomes of a lagged and with respect to time integrated process of the traffic system.

We therefore use smoothed interpolated values of vehicle kilometers and smoothed interpolated values of differences for the calculated approximations. Since our main interest lies in the "prediction" of long-term developments in fatalities from the growth in vehicle kilometers smoothed interpolated values also will serve our macroscopic approach.

Let the smoothing of vehicle kilometers be performed by Newtonian interpolation as

$$\hat{V}_t = \sum_{n=-1}^{n=1} w_n V_{t+n} \qquad (41)$$

where $\Sigma w_n = 1$ and w_n is decreasing backward and forward by the binomial reciprocal of n. For our analyses we have chosen Newtonian interpolation with l=3; however any other well-established smoothing method would have served our purpose as well.

A quite accurate approximation of the value of the derivative is given by Stirling's interpolation of central differences as:

$$V_t^* \approx \triangle V_t = 1/k\{(\hat{V}_{t+k} - \hat{V}_{t-k})/2 - 1/6(\hat{V}_{t+2k} - \hat{V}_{t-2k})/4\}$$

where we choose k=3. Again we smooth by identical interpolation as in (41)

$$\triangle\hat{V}_t = \sum_{m=-i}^{m=i} w_m \triangle V_{t+m} \qquad (42)$$

Here we choose i=5 because of the larger apparent irregularity of $\triangle V_t$. In order not to lose too many values at the end and beginning of the series, we used also some forward and backward extrapolations for V_{t-j} and V_{t+j}, where j ranges from 1 to k+i. Because of the exponentional nature of the growth curves we used second order Newtonian extrapolation on the logarithmic values of V_t.
For the above methods of smoothing, extrapolation and approximation we refer to standard textbooks on the calculus of finite differences.

Substituting (41) and (42) into (11) we obtain

$$Q_{t'} \approx \hat{Q}_{t'} = \frac{\triangle\hat{V}_{t'}}{\hat{V}_{t'}} \qquad (43)$$

The variables of (41), (42) and (43) as such can be used as observed variables in the equations of (37) to (40), since no estimation of any parameter was involved in their calculations.

Now, apart from the transformation of vehicle kilometers to exposure, we are ready to plot all the relevant pairs of variables for several countries against the time axes in order to inspect the validity of our assumptions.

6.2. Federal Republic of Germany

Figure 10 plots the vehicle kilometers (defined by (41)) and the increment in vehicle kilometers (defined by (42)) for the FRG from 1953 to 1985.

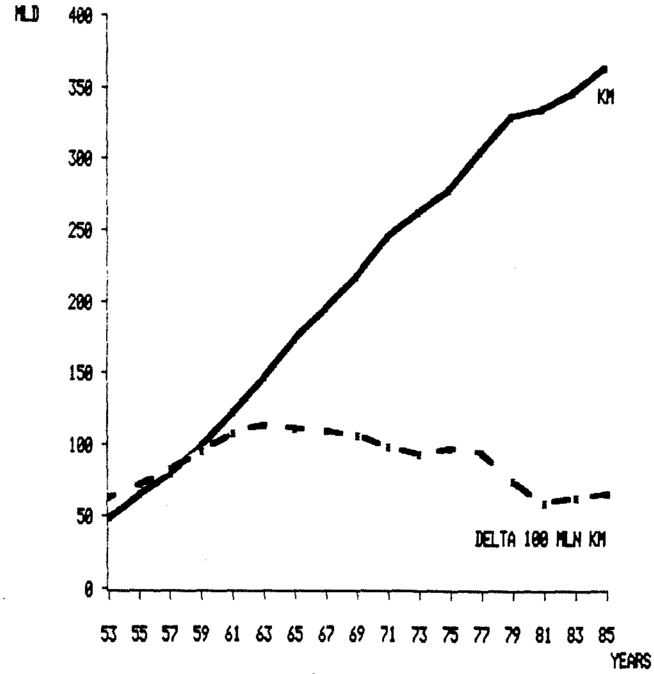

Figure 10. Growth and increase of veh. km. in the FRG.

We see from the development of increments that the hypothesis of saturation of growth is not falsified, although the rise at the end may cast some doubts. Surely economic fluctuations (1974 oil-crisis and 1981 deepest point of recession) may form an additional explanation.

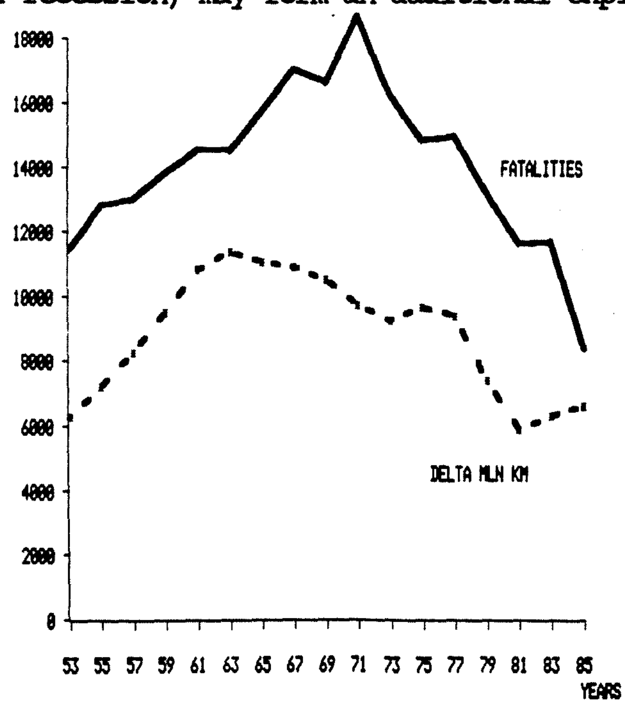

Figure 11. Increase of veh. km. and fatalities in the FRG.

In Figure 11 we present the developments of fatalities and again of increments in vehicle kilometers. The figure reveals a remarkable overall resemblance in development. As predicted from our adaptive system theory, the apparent shift for fatalities with respect to increment of vehicle kilometers, indicates a time-lag. The time-lag for fatalities seems to be about 9 years. The coinciding lagged development of fatalities and increase of vehicle kilometers seems to sustain the simplified specific assumption of (40b).
Since growth in vehicle kilometers in the FRG is definitely not exponentional, this points to a lagged enlargement of roads in the FRG. The existence of a time-lag also suggests that proportional decrease in fatality rate and acceleration is not valid. The nearly proportional relation between fatalities and increments seems to sustain the hypothesis of equal speeds of growth and adaptation and the simplifications by the equivalence of power-parameters in the equations of growth and adaptation.

Finally, in Figure 12 we plot the fatality rate (defined in (25a) for s=1) and the acceleration against time.

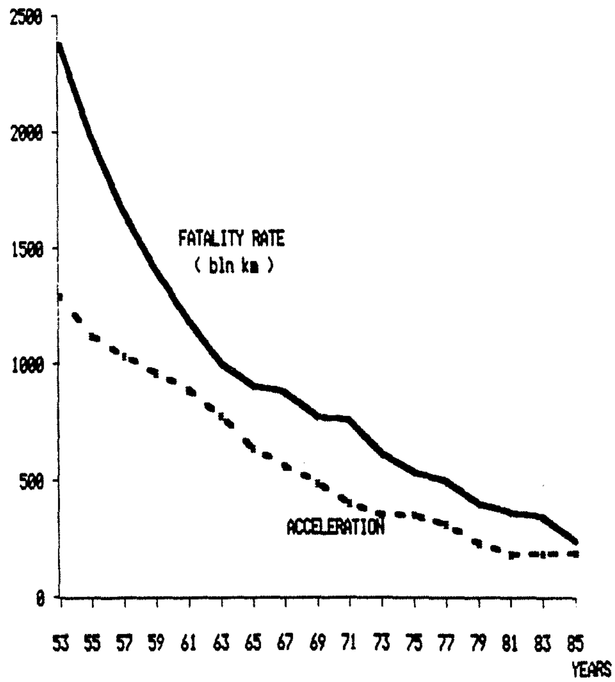

Figure 12. Fatality rate and acceleration in the FRG.

The shapes of the curves are quite well in agreement with some of the mathematically hypothesized curves, illustrated in Figures 8 and 9. The logistic type of curves (beta-model and logistic growth) is only applicable if the inflexion point lies before or around the start of the time-series available. Since exponentional decrease is in conflict with

(40b), while Figure 11 agrees with (40b), the linear-operator model and Gompertz growth are not likely applicable. Therefore, generalized log-reciprocal growth and adaptation along the generalized urn-model seems most likely. Remembering that the time-lag in Figure 11 was about 9 years, the resemblance of the shifted curves strongly supports the basic assumption of (37) developed from the adaptive system theory.

In conclusion, we see the case of the FRG as a nice illustration of the validity of the general theory. For the FRG, moreover, some conditions for the simplification of at least (39b) are fulfilled while the additional condition for (40b) is quite well approximated. If the theory is true and has predictive power, the time-lag enables one to predict a stagnation in the drop of fatality rate in the nineties due to the almost increasing acceleration curve after 1981 in FRG.

6.3. <u>France</u>

For France the data from 1960 to 1984 are plotted in the same way as for the FRG in Figures 13 and 14.

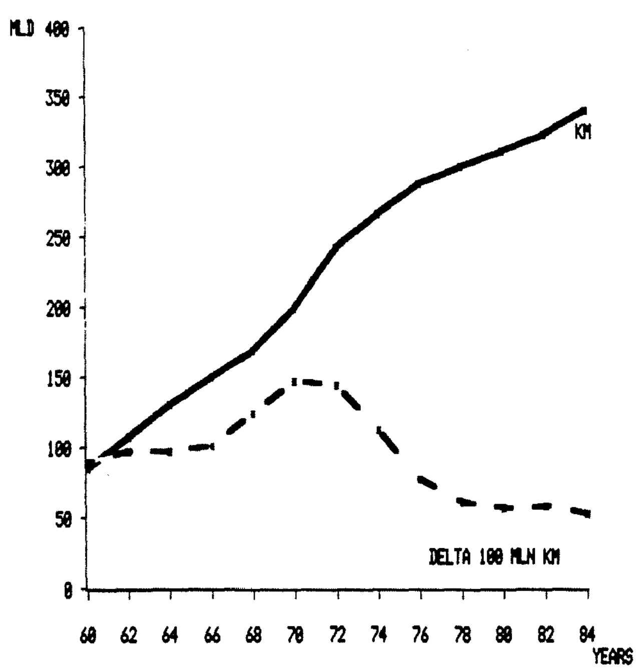

Figure 13. Growth and increase of veh. km. in France.

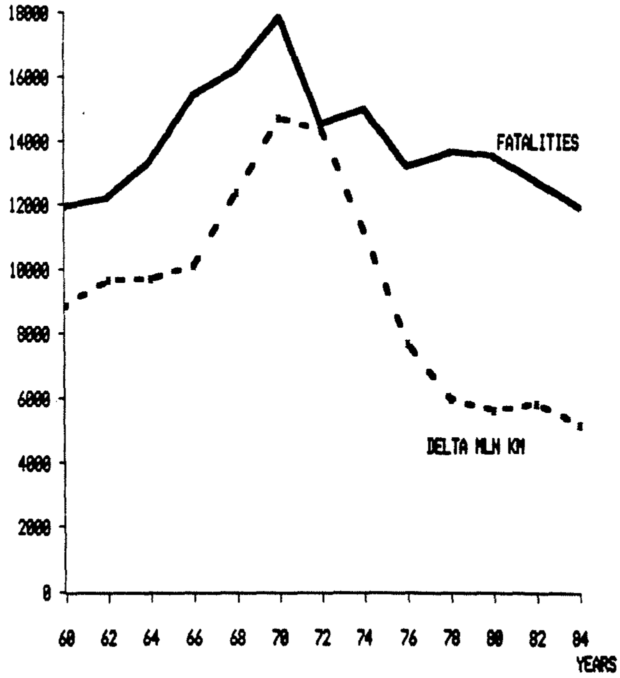

Figure 14. Increase of veh. km. and fatalities in France.

From Figure 13 we see that the sigmoid growth curve for vehicle kilometers is a well-suited assumption. Figure 14 shows a fair correspondence in curves, but does not show a time-lag. This is, however, quite in agreement with the simplified specific assumption of (40a) or rather with the specific assumption of (39a) and surely with the simplified basic assumption of (38a), since in the latter case by curve-fitting, we may achieve a better correspondence for the recent 10 years in Figure 14. In Figure 15 we plot fatality rate (again defined in (25a) for s=1) and acceleration against time. With regard to the small irregularities of the curves for fatality rate and fatalities one has to bare in mind that no smoothing of these curves was performed.

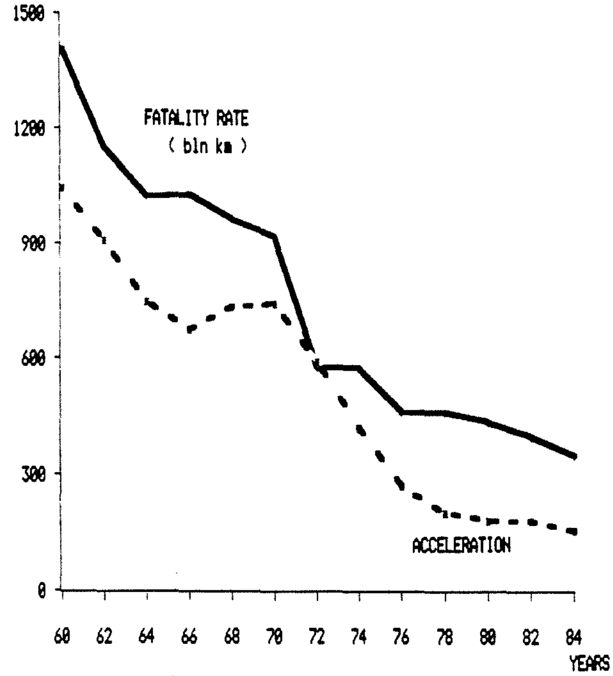

Figure 15. Fatality rate and acceleration in France.

The most striking aspect of Figure 15 is the marked divergence from the monotonically decreasing functions illustrated in Figures 8 and 9, while the correspondence between the plotted curves in Figure 15 remains apparently intact. This common departure seems to justify the conjecture that the relation between adaptation and growth expressed in the basic assumption of (37) will hold irrespective of the functions by which adaptation and growth are expressed. Since at least (38a) explains the results from Figure 14, while Figure 15 clearly violates a proportional decrease, the simplification for (38a) in case of France must be found either in the absence of a time-lag for adaptation or in corresponding departures from proportionality. In our opinion the latter option seems most likely in view of the common departure from the hypothesized curves. Because of the non-steady decrease in the plotted curves no particular type of curve will fit nicely, although the corresponding departure from proportionality points to the Gompertz curve for growth and the linear-

operator model for adaptation. From a macroscopic point of view one may judge this satisfactory.
We conclude that the validity of the general theory is fairly well illustrated by the data from France since certainly (37) holds. Moreover, at least some of the conditions for the simplified basic assumption of (38a) seem to be fulfilled and the additional condition for the specific assumption of (39a) is approximated.

6.4. The Netherlands

For the Netherlands the data from 1950 to 1986 are plotted in Figures 16 and 17 in the same way as before.

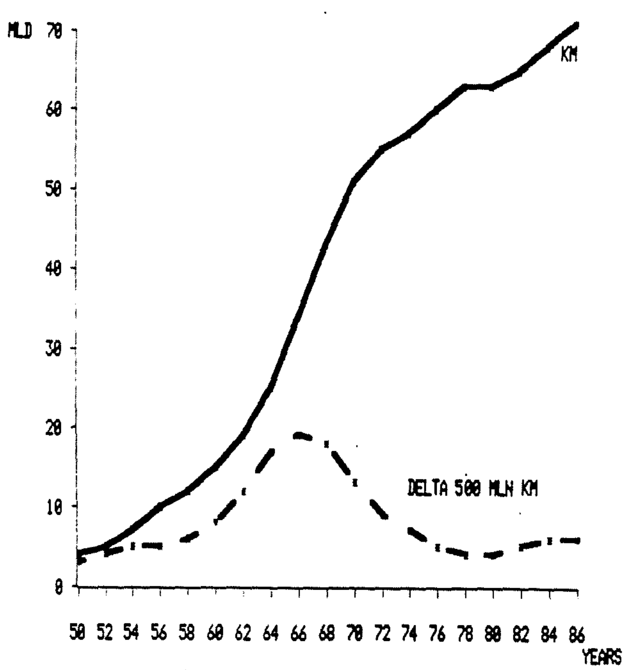

Figure 16. Growth and increase of veh. km. in the Netherlands

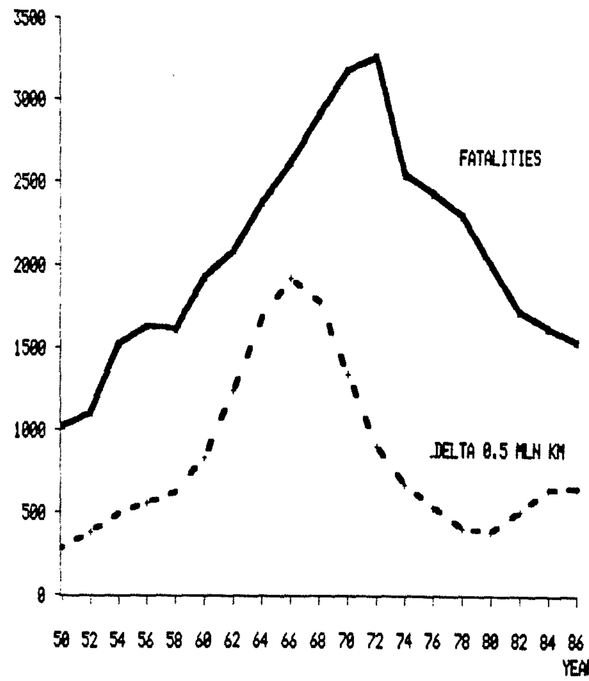

Figure 17. Increase of veh. km. fatalities in the Netherlands

Again the sigmoid growth curve appears; as in the case of the FRG some caution with regard to the contradictory increasing increments in the latest years is in place. Again the economic recession with its deepest point in 1981 may be an additional explanation for temporary departures. Figure 17 shows again a remarkable resemblance in the development of fatalities and increase of vehicle kilometers. There is an apparent time-lag of about 6 years. This strongly supports the applicability of (40b) and possibly also the validity of some conditions that lead to that simplification. Surprisingly, this does not imply a close resemblance of curves for fatality rate and acceleration as exhibited in Figure 18.

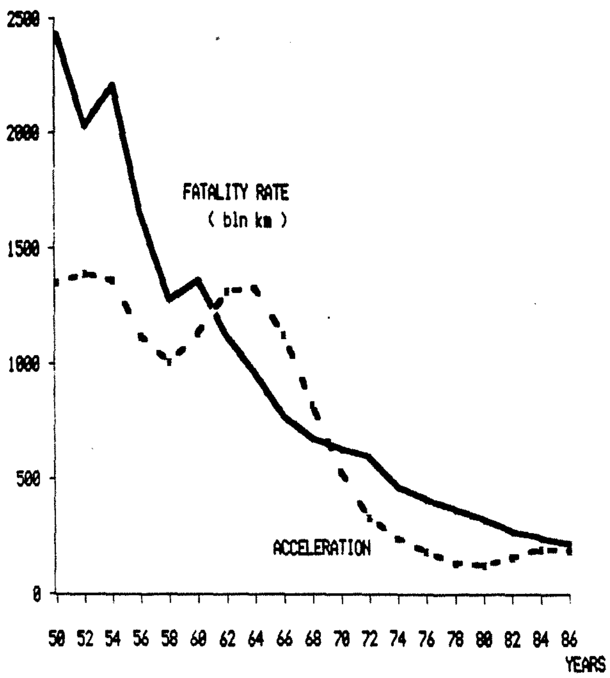

Figure 18. Fatality rate and acceleration in the Netherlands.

The fatality rate could follow any adaptation model. The acceleration curve shows a remote resemblance to a logistic curve. Only if adaptation is of the same type the basic assumption will hold. Since this assumption is supported we have to investigate the beta model for adaptation as well. Assuming only (39b), we may transform these curves by a free power-parameter. As an illustration Figure 19 shows the square root of the acceleration shifted by 6 years and the fatality probability as rate by exposure measured by the square root of vehicle kilometers.

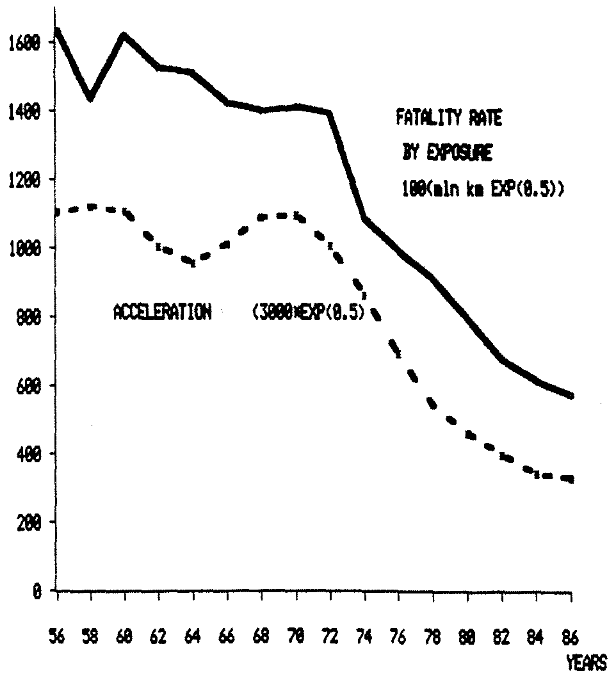

Figure 19. Transformed curves of rates for the Netherlands.

Doing so we kept the condition for (39) intact since $s = \mu = 1/2$. Both curves are clearly of the logistic type and the predicted close correspondence is restored and could even be improved by a somewhat smaller time-lag than 6 years.

For the Netherlands we conclude also that the illustrative results sustain the validity of the general theory since the basic assumption of (37) surely applies. Moreover, the specific assumption of (39b) seems to be justified.

6.5. Great Britain

The data for Great Britain from 1950 to 1984 are shown as before in Figures 20 and 21.

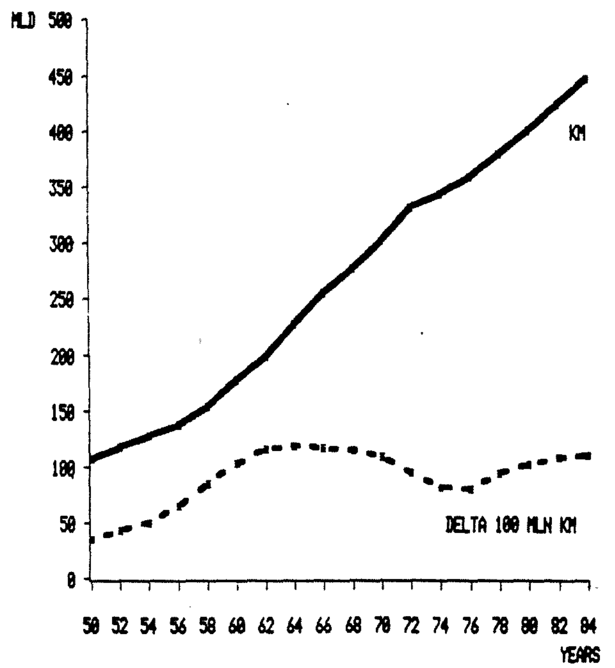

Figure 20. Growth and increase of veh. km. in Great Britain

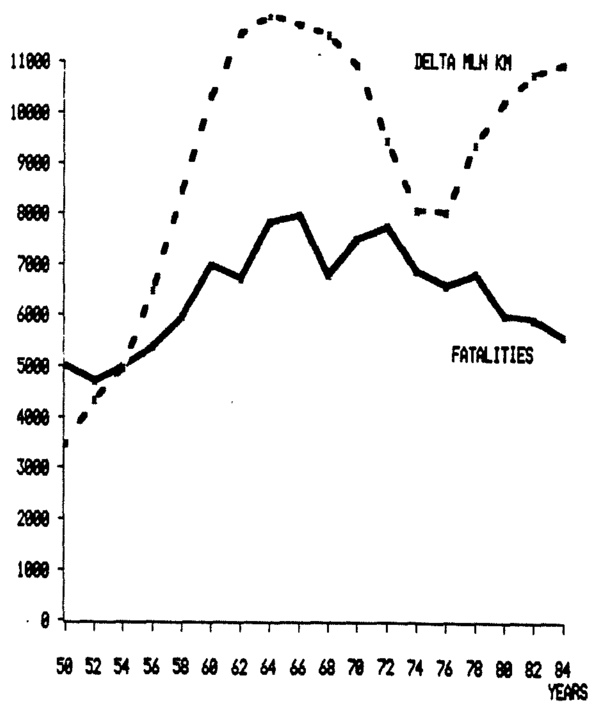

Figure 21. Increase of veh. km. and fatalities in Great Britain

From the curve for the increase in vehicle kilometers we see that the hypothesized sigmoid curve with saturation is violated by the incremental increase after 1976. Moreover, there is no close resemblance in development of fatalities and increase in vehicle kilometers. Clearly the equivalence condition for (39a) in contrast to other countries is not satisfied . In Figure 22 we show the curves for fatality rate and acceleration in the usual way, while in Figure 23 without optimization we illustrate these curves after theoretically allowable transformations.

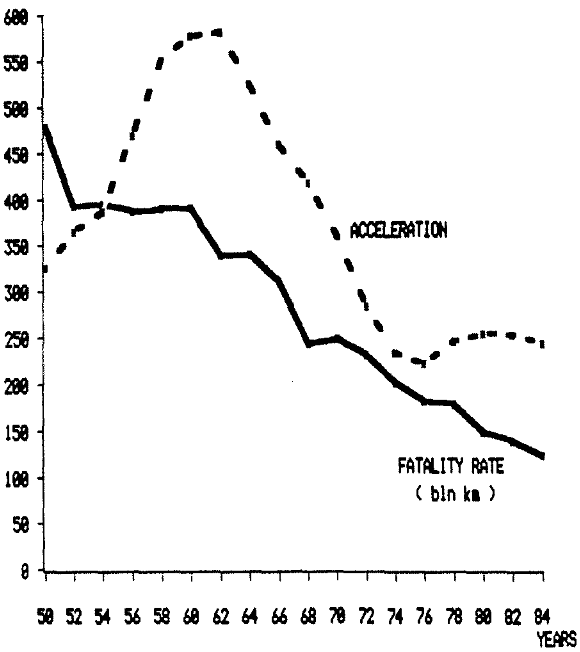

Figure 22. Fatality rate and acceleration in Great Britain

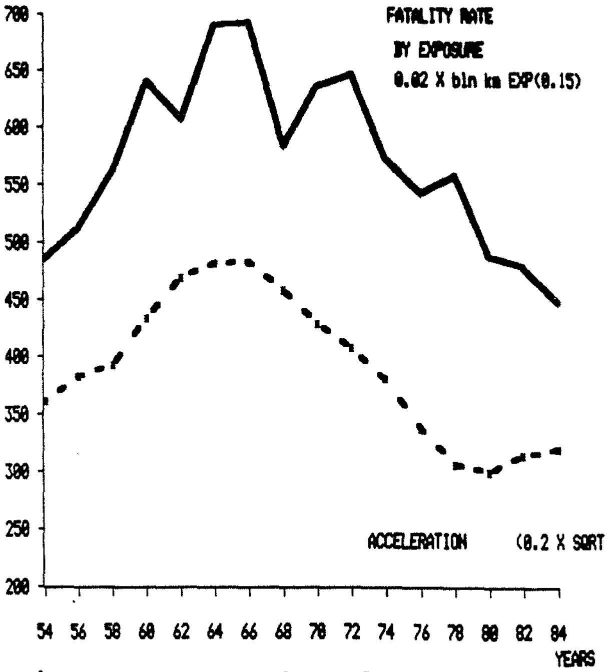

Figure 23. Transformed curves of rates for Great Britain

The fatality rate of Figure 22 confirms the adaptation models; the acceleration, however, violates the proposed growth models. Apparently Figure 23 still sustains the basic assumption of (37b) in a macroscopic sense (bare in mind there is no smoothing for fatality-rate curve) and there seems to be a time-lag of less than 4 years. The shape of these curves is not of the predicted decreasing type and thereby violates the interpretation given in the theory. Apparently growth and acceleration behave not as predicted in the case of Great Britain. However, some caution is necessary since the recorded vehicle kilometers include falling bicycle kilometers in the post war-period too. Although conceptually Figure 23 is not well comprehensible, the mathematical expression for the basic assumption of (37) still seems to hold. Therefore, we may see Great Britain also as a justification of the conjecture that the basic assumption of (37) mathematically holds irrespective of the type of functions for growth or adaptation. The general theory with respect to growth, however, is not well supported in the case of Great Britain.

6.6. United States of America

For the USA we have the longest series of data, from 1933 to 1985. In Figures 24 and 25 we display these data in the usual way.

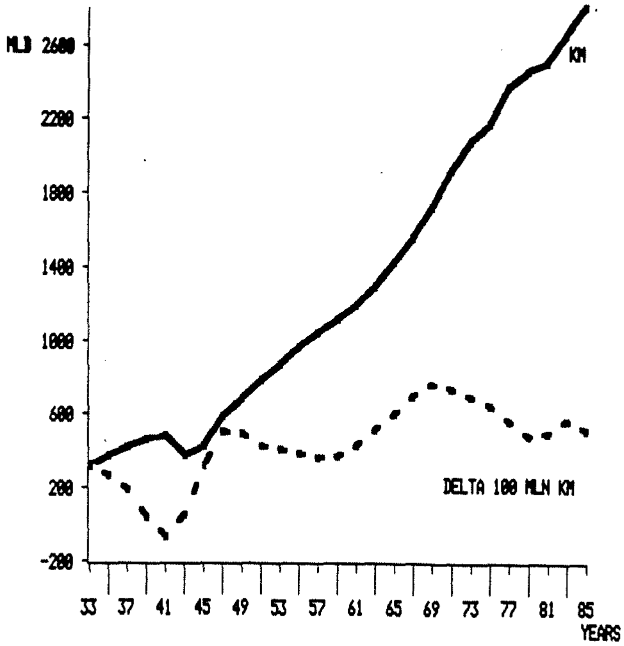

Figure 24. Growth and increase of veh. km. in the USA.

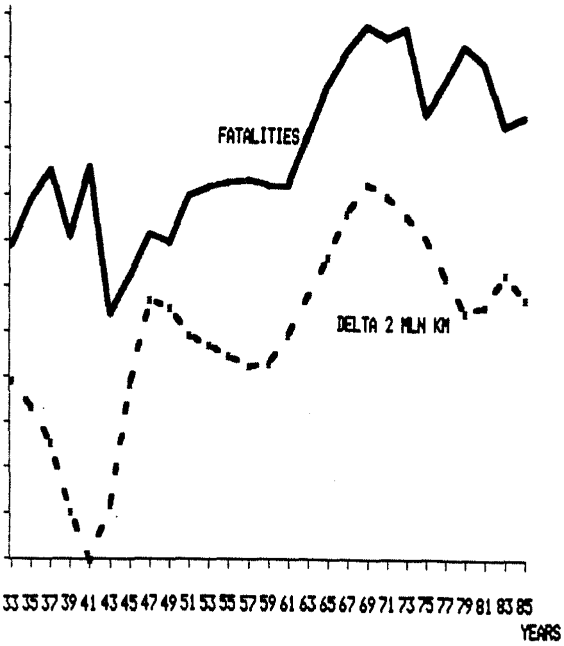

Figure 25. Increase of veh. km. and fatalities in the USA.

Even ignoring the war-period the increase of the vehicle kilometers does not show a clear sigmoid curve. Despite this non-saturating growth we see from Figure 25 after the war a macroscopic resemblance in the development of fatalities and increase of vehicle kilometers. There is no apparent time-lag. This sustains the simplified specific assumption of (40a) and makes a proportional adaptation and or acceleration probable.
Finally, in Figure 26 we plot again acceleration and fatality rate.

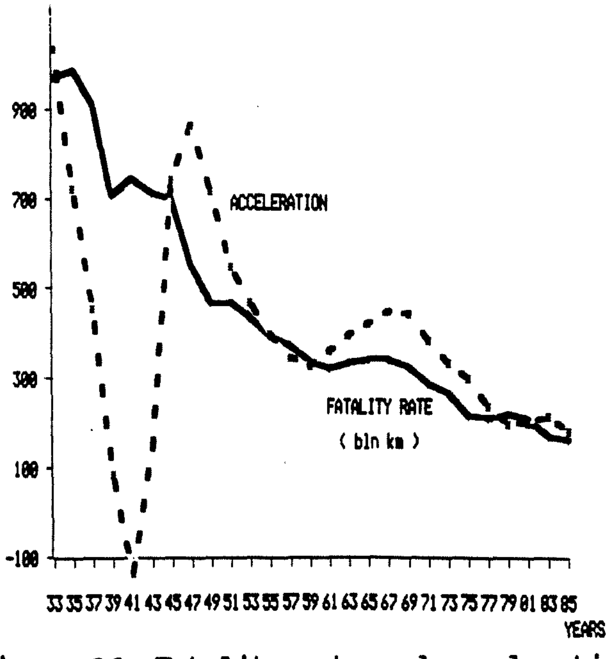

Figure 26. Fatality rate and acceleration in the USA.

Again we see a remarkable correspondence between both curves after, say 1946. This common curvature after 1946 can even be improved, flattening the acceleration curve somewhat more than the fatality rate by taking both the power-parameters μ and s somewhat below unity. Thereby, we fall back on the specific assumption of (39a) keeping $s = \mu$ as the condition for this assumption intact. As was already implied by the absence of a time-lag some proportionality has to be the case; we see from the fatality rate that this may be quite appropriate since the linear-operator model for adaptation could be satisfied very well. The sharp drop for the acceleration in the war-period to even negative values indicates that temporary external influence on growth, without disturbing the total system, has no direct effect on the process of adaptation. This can be seen as justification for the conjecture that adaptation is a lagged and over many years integrated process.

In conclusion, we take the case of the USA as an indication for the validity of our general theory since the basic assumption certainly holds. Moreover, at least some sufficient conditions that lead to the specific assumption of (39) are fulfilled in the case of the USA.

7. EXTENDED ANALYTICAL CONSIDERATIONS

. From (37) and (43) we write by taking logarithm

$$\ln F_t = \ln \delta + \mu \ln \hat{Q}_{t'} + s \ln V_t \qquad (44)$$

. This can be fitted by ordinary multiple regression for different time-lags of t-t` in order to find optimal parameters. One can also find similar ways for the optimal fitting procedures for curves of growth and adaptation. It also could be shown that by alternating least-squares procedures a fitting procedure for the non-diagonal cases of Table 1 can be developed in order to find optimal parameters and to select the optimal models.

The statistical and numerical analyses will be presented elsewhere (Koornstra, 1989 forthcoming). One very interesting extension of the theory already outlined by Koornstra (in Oppe et al., 1988) and more fully to be presented in the forthcoming publication, is mentioned here as the general basic assumption.

Let the number of any type of negative outcomes of traffic events between pure encounters and fatalities, divided by exposure be defined as R_t. Then the general basic assumption states that this rate, for example the injury rate, is a sum of a constant π and the with $(1-\pi)$ weighted fatality rate as defined by (37).

. This is written as

. general basic assumption

$$R_t = \pi + (1-\pi) \; \{ \; \delta \, Q^{\mu}_{t'} \} \qquad (45)$$

. Substituting the expressions for Q^{μ}_{t}, from (12), (13) and (14) into (45) we obtain apart from the time-lag the generalized adaptation models of (21), (22) and (23) for $1 > \pi > 0$. Clearly for exposure itself $\pi=1$ and for fatalities $\pi=0$.

This states that at the end of the growth process when the increase in vehicle kilometers is zero due to saturation, the fatality rate should reduce to zero too. This is quite in agreement with the just shown results where the proportional relation between acceleration and fatality rate was validated. In contrast to fatality rate the rates for less severe outcomes of accidents will not reduce to zero, but to a constant according to (45). Applying the simplifications made before on fatalities it turns out that the development of such quantities as the number of injuries is described by a weighted sum of vehicle kilometers and the increase in vehicle kilometers. We do not develop this matter further here, but we show, merely as an example, the observed injury rate in the Netherlands (injuries defined as being at least one day in the hospital) in Figure 27.

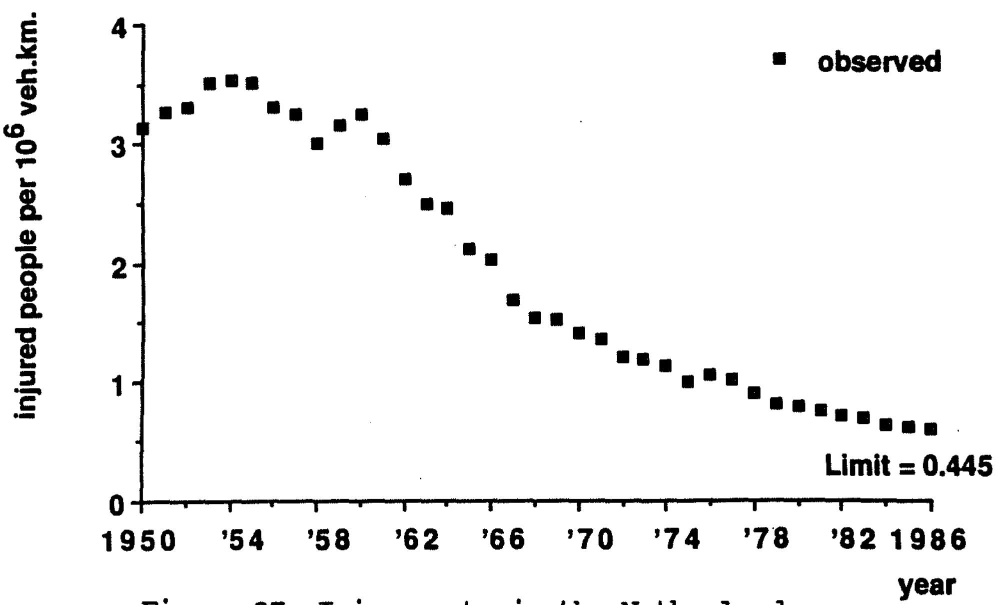

Figure 27. Injury rate in the Netherlands.

Clearly a logistic type of curve is present. Therefore, we fitted the generalized beta-model of (21) as the adaptation model in place and find the optimal parameter for $\pi = 0.445$. So at least there is some validity for the general basic assumption of our theory given in (45). It will be noted that the development of outcomes of events between mere encounters and fatal accidents are in the general case of (45) an additive function of the development of (power-transformed) vehicle kilometers and the product of (power-transformed) vehicle kilometers and their (power-transformed) acceleration.

8. CONCLUSIONS

-I-

The developed mathematical theory of self-organizing adaptive systems applied to traffic states that

- the development of fatality rate is a simple mathematical function of the rate of increase in vehicle kilometers.

Some plausible simplifications reduces this statement to

- the development of fatalities is proportional to the increase in vehicle kilometers.

The latter was demonstrated to be approximately the case for data from the Federal Republic of Germany, France, the Netherlands and the United States of America. The former applies to data from Great Britain.
The time-series of data ranged from 25 years (France) to 53 years (USA).
The validation holds for long-term trends in the developments.
The theory predicts a fatality rate reduction to near zero. This reduction to near zero is not predicted for rates of less severe outcomes of accidents.

-II-

Comparison of the fatality-rate curve and the curve for rate of increase of growth in vehicle kilometers, with respect to overall level and overall steepness of descent of these curves for the mentioned countries, reveals:

- a perfect rank-order correlation between the levels of both curves (high = France -> FRG -> Netherlands -> USA -> Great Britain = low)
- a nearly perfect rank-order correlation between steepness of descent in both curves (flat = Great Britain ≈ USA -> France -> FRG -> Netherlands = steep)

and subsequently

- a moderate negative rank-order correlation between level and steepness of descent of the fatality-rate curve.

-III-

The above summarized findings support the proposed theory of adaptive self-organizing systems with respect to the emergence of traffic safety. If this theory is correct it follows that the best policy for safety is:

- A controlled moderate growth of traffic leading to a reduced rate of increase for growth of vehicle kilometers, which in turn leads to a lower total number of fatalities.
- Analogous to mutations in biological systems: enhancement of variety and creativity in safety measures (possibly by decentralization and planned experimentation as well as creative research).
- Analogous to selection in biological systems: objective long-term evaluation of effects and selection of effective safety measures.
- Replication of effective safety measures in other places and domains.

-IV-

The last part of conclusion -II- and conclusion -III- point to the fact that adaptation in the self-organizing traffic system is not an automatic, even if possibly an autonomic, process in society. Unlike biological self-organizing systems, adaptation is governed by decision-making bodies and individuals and their decisions do matter.

9. LITERATURE

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