The basic evaluation model

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Summary

This report provides a detailed technical description of the proposed methodology for the evaluation of developments in Dutch road safety in the SWOV projects ‘Verkeersveiligheidsbalansen’ (Road safety assessments) and ‘Verkeersveiligheidsverkenningen’ (Road safety outlooks). The methodology is based on structural time series models by state space methods, in order to accommodate the dependencies between observations consisting of repeated measurements over time.
A dedicated bivariate local linear model, called the basic evaluation model, is proposed, in which the development of road safety is assumed to be the product of the developments of two latent, unobserved variables: exposure and risk.
The methodology is illustrated with a simultaneous analysis of the annual number of Dutch road traffic fatalities and motor vehicle kilometres driven in the period 1948-1998. The results are found to compare favorably with those obtained with standard analysis techniques, and the report concludes with a number of possible extensions of the basic evaluation model.

Contents

1. Introduction

This report provides a detailed technical description of the proposed methodology for the evaluation of developments in Dutch road safety in the SWOV projects ‘Verkeersveiligheidsbalansen’ (Road safety assessments) and ‘Verkeersveiligheidsverkenningen’ (Road safety outlooks), as has already been outlined in Bijleveld (1999).

The objective of the project Road safety assessments is to model and obtain explanations for the development of road traffic safety in the Netherlands, while the project Road safety outlooks is aimed at obtaining forecasts for the future development of Dutch road safety.
In both projects, the variables which are considered typically consist of observations repeatedly measured over time. Usually, such observations are not independent of one another. As an example, consider the annual number of crashes resulting in a person being killed, as recorded over a number of years. Although individual accidents are independent of one another over the years, their annual totals are not, being the result of a traffic process that slowly evolves over time. Therefore, the number of crashes that happen in a certain year, is often quite a good predictor for the number of accidents that will occur in the following year.
When the dependencies between the observations are ignored, and classical linear or non-linear regression models are used for the analysis of developments over time, these dependencies usually cause the residuals of such analyses also to be serially correlated. Since the standard errors required for the calculation of confidence intervals and of statistical tests in such models are based on the assumption of independence of the residuals, the values of these statistics and therefore the conclusions drawn from the analyses may be incorrect.

In this report we propose to analyze the developments in road safety with structural time series models by state space methods (Harvey, 1989; Durbin & Koopman, 2001). These time series analysis models explicitly deal with the dependencies in the data, thereby usually yielding residuals that do satisfy the assumption of independence. A dedicated multivariate state space model, called the basic evaluation model, is presented, in which the development of traffic safety is assumed to be the product of the developments of two latent, unobserved variables: exposure and risk. Exposure is treated as a latent variable because the mobility figures –which act as an indicator variable for the unobserved exposure- are assumed to be subject to measurement error. The reason that the risk is modelled as a latent variable is that this is indeed what risk is: an unobservable entity which can not be measured directly.

The basic evaluation model is illustrated with the simultaneous analysis of the total number of annual traffic fatalities in the Netherlands in the period 1948-1998 and the total number of annual motor vehicle kilometres driven. The results of the analysis are compared with those obtained with classical linear regression. The report concludes with a number of possible extensions of the proposed methodology.

2. The basic evaluation model

The basic evaluation model is a special case of state space methods for the analysis of time series (Harvey, 1989; Durbin & Koopman, 2001). In matrix algebra, all state methods can very generally be expressed as

$$y_t = Z_t \alpha_t + \varepsilon_t, \qquad \varepsilon_t \sim NID(0, H_t) \tag{1}$$

$$\alpha_{t+1} = T_t \alpha_t + R_t \eta_t, \qquad \eta_t \sim NID(0, Q_t) \tag{2}$$

for t = 1, ..., n, where $\varepsilon_t \sim NID(0,H_t)$ is a short-hand notation for: the errors or disturbances ε_t are assumed to be normally and independently distributed with means equal to zero and a variance structure equal to H_t.
The basic model for evaluating the developments in Dutch road traffic safety is a bivariate local linear trend model. Specifically, let

$$y_t = \begin{pmatrix} y_t^{(1)} \\ y_t^{(2)} \end{pmatrix} = \begin{pmatrix} \log M_t \\ \log F_t \end{pmatrix},$$

where M_t are the observed mobility figures and F_t are the observed figures for fatal accidents, road traffic fatalities or serious injuries at time points t = 1, ..., n.
Also, temporarily assuming annual figures (and thus ignoring seasonal patterns), define

$$\alpha_t = \begin{pmatrix} \mu_t^{(1)} \\ \nu_t^{(1)} \\ \mu_t^{(2)} \\ \nu_t^{(2)} \end{pmatrix}, \quad \eta_t = \begin{pmatrix} \xi_t^{(1)} \\ \zeta_t^{(1)} \\ \xi_t^{(2)} \\ \zeta_t^{(2)} \end{pmatrix}, \quad T = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix},$$

$$H_t = \begin{bmatrix} \sigma^2_{\varepsilon^{(1)}} & \mathrm{cov}(\varepsilon^{(1)}, \varepsilon^{(2)}) \\ \mathrm{cov}(\varepsilon^{(1)}, \varepsilon^{(2)}) & \sigma^2_{\varepsilon^{(2)}} \end{bmatrix}, \text{ and}$$

$$
Q_t = \begin{bmatrix} \sigma^2_{\xi^{(1)}} & 0 & \mathrm{cov}(\xi^{(1)},\xi^{(2)}) & 0 \\ 0 & \sigma^2_{\zeta^{(1)}} & 0 & \mathrm{cov}(\zeta^{(1)},\zeta^{(2)}) \\ \mathrm{cov}(\xi^{(1)},\xi^{(2)}) & 0 & \sigma^2_{\xi^{(2)}} & 0 \\ 0 & \mathrm{cov}(\zeta^{(1)},\zeta^{(2)}) & 0 & \sigma^2_{\zeta^{(2)}} \end{bmatrix}
$$

Writing out (1) in scalar notation yields the following two observation equations:

$$
\begin{aligned} y_t^{(1)} &= \mu_t^{(1)} + \varepsilon_t^{(1)} \\ y_t^{(2)} &= \mu_t^{(1)} + \mu_t^{(2)} + \varepsilon_t^{(2)} \end{aligned}, \tag{3}
$$

while working out (2) in scalar notation results in the following four state equations:

$$
\begin{aligned} \mu_{t+1}^{(1)} &= \mu_t^{(1)} + \nu_t^{(1)} + \xi_t^{(1)} \\ \nu_{t+1}^{(1)} &= \nu_t^{(1)} + \zeta_t^{(1)} \\ \mu_{t+1}^{(2)} &= \mu_t^{(2)} + \nu_t^{(2)} + \xi_t^{(2)} \\ \nu_{t+1}^{(2)} &= \nu_t^{(2)} + \zeta_t^{(2)} \end{aligned}. \tag{4}
$$

Since $\mu_t^{(1)}$ and $\mu_t^{(2)}$ are the trends for the exposure and the risk, respectively, and the mobility and fatality figures are modelled in their logarithms, the second equation in (3) can be written as

$$
\log F_t = \log(\text{trend Exposure}) + \log(\text{trend Risk}) + \log(\text{error}) \tag{5}
$$

and therefore as

$$
\log F_t = \log[(\text{trend Exposure})(\text{trend Risk})(\text{error})], \tag{6}
$$

since $\log a + \log b = \log(ab)$.
Finally, taking the exponent of (6) yields the following multiplicative model

$$
\text{Traffic Safety} = (\text{trend Exposure})(\text{trend Risk})(\text{error}) . \tag{7}
$$

When all disturbances in (4) are fixed on zero, the basic model collapses to a bivariate linear regression model:

$$
\begin{aligned} \log M_t &= a^{(1)} + b^{(1)}t + \varepsilon_t^{(1)} \\ \log F_t &= a^{(1)} + b^{(1)}t + a^{(2)} + b^{(2)}t + \varepsilon_t^{(2)} \end{aligned} \tag{8}
$$

with $t = 1, \ldots, n$.

3. Illustration

As an illustration, the basic evaluation model was applied to the log of the Dutch annual figures of road traffic fatalities and to the log of the Dutch motor vehicle kilometres driven (mvkms) in the period 1948-1998. All analyses were performed in Ox (Doornik, 2001) and SsfPack (Koopman, Shephard, and Doornik, 1999).
For these two time series, the estimated variances of the level disturbances in (4) are

$$\begin{bmatrix} \sigma^2_{\xi^{(1)}} & \operatorname{cov}(\xi^{(1)}, \xi^{(2)}) \\ \operatorname{cov}(\xi^{(1)}, \xi^{(2)}) & \sigma^2_{\xi^{(2)}} \end{bmatrix} = \begin{bmatrix} 0.00032978 & -0.00052381 \\ -0.00052381 & 0.0034489 \end{bmatrix}$$

showing that, since the covariance between the two level disturbances is negative, the trends in exposure and risk are negatively related (as expected), while those of the slope disturbances in (4) equal

$$\begin{bmatrix} \sigma^2_{\zeta^{(1)}} & \operatorname{cov}(\zeta^{(1)}, \zeta^{(2)}) \\ \operatorname{cov}(\zeta^{(1)}, \zeta^{(2)}) & \sigma^2_{\zeta^{(2)}} \end{bmatrix} = \begin{bmatrix} 0.00016127 & 0.00005830 \\ 0.00005830 & 0.00002108 \end{bmatrix}$$

The estimated variances of the observation disturbances in (3) are

$$H_t = \begin{bmatrix} \sigma^2_{\varepsilon^{(1)}} & \operatorname{cov}(\varepsilon^{(1)}, \varepsilon^{(2)}) \\ \operatorname{cov}(\varepsilon^{(1)}, \varepsilon^{(2)}) & \sigma^2_{\varepsilon^{(2)}} \end{bmatrix} = \begin{bmatrix} 0.00000806 & 0.00008647 \\ 0.00008647 & 0.00093741 \end{bmatrix}$$

At convergence of the algorithm, the value of the log-likelihood for this model is 164.79, and the value of the Akaike Information Criterion (AIC) equals -2.9370.

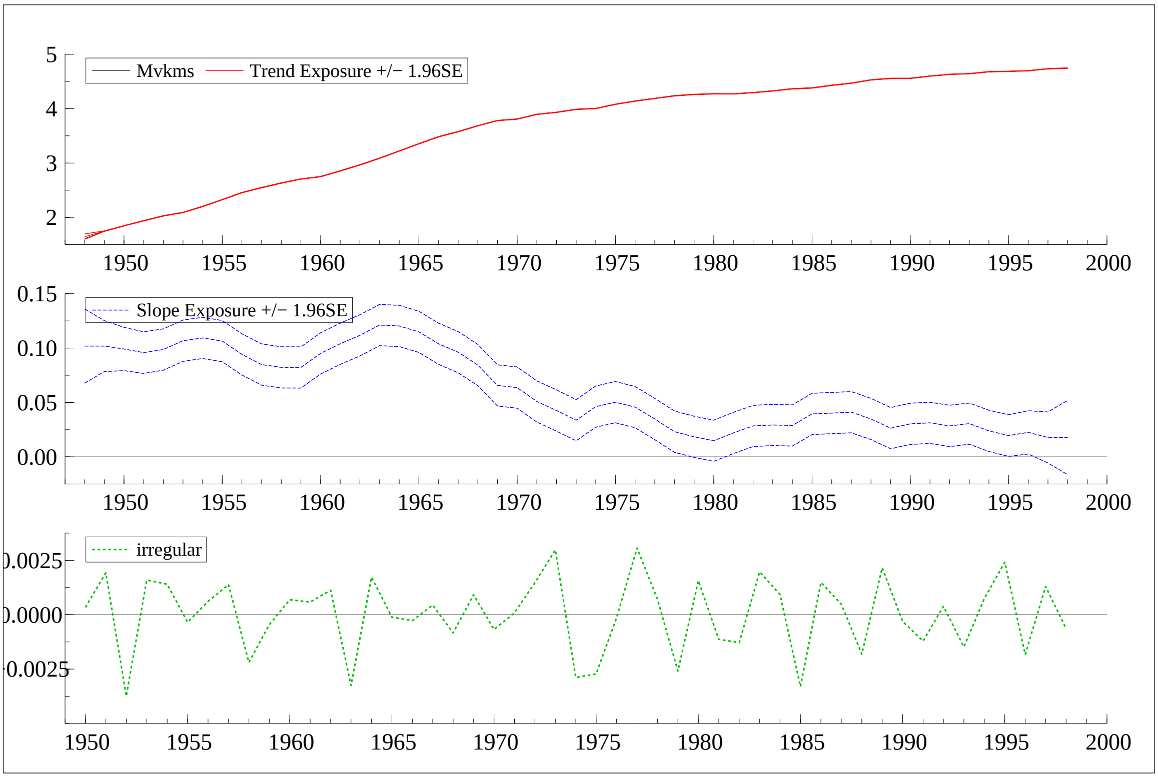

Figure 1. *Dutch development in exposure in the years 1948-1998: trend, slope and irregular component*.

Plots of the results of the analysis are displayed in *Figures 1* to *3*. The top graph in *Figure 1* shows the log of the Dutch annual mobility figures, together with the modelled development or trend of the exposure $\mu_t^{(1)}$ (corresponding to the first equation in (4)), including its 95% confidence interval. The middle graph displays the development of the slope component $\nu_t^{(1)}$ for the exposure (the second equation in (4)), including its 95% confidence interval, while the bottom graph with the legend 'irregular' contains the observation disturbances or residuals $\varepsilon_t^{(1)}$ corresponding to the first equation in (3).
Since the value of the slope component in *Figure 1* is positive throughout the observed period, the exposure was increasing all the time, albeit at varying rates: the rate of increase being larger at the beginning than at the end of the period.

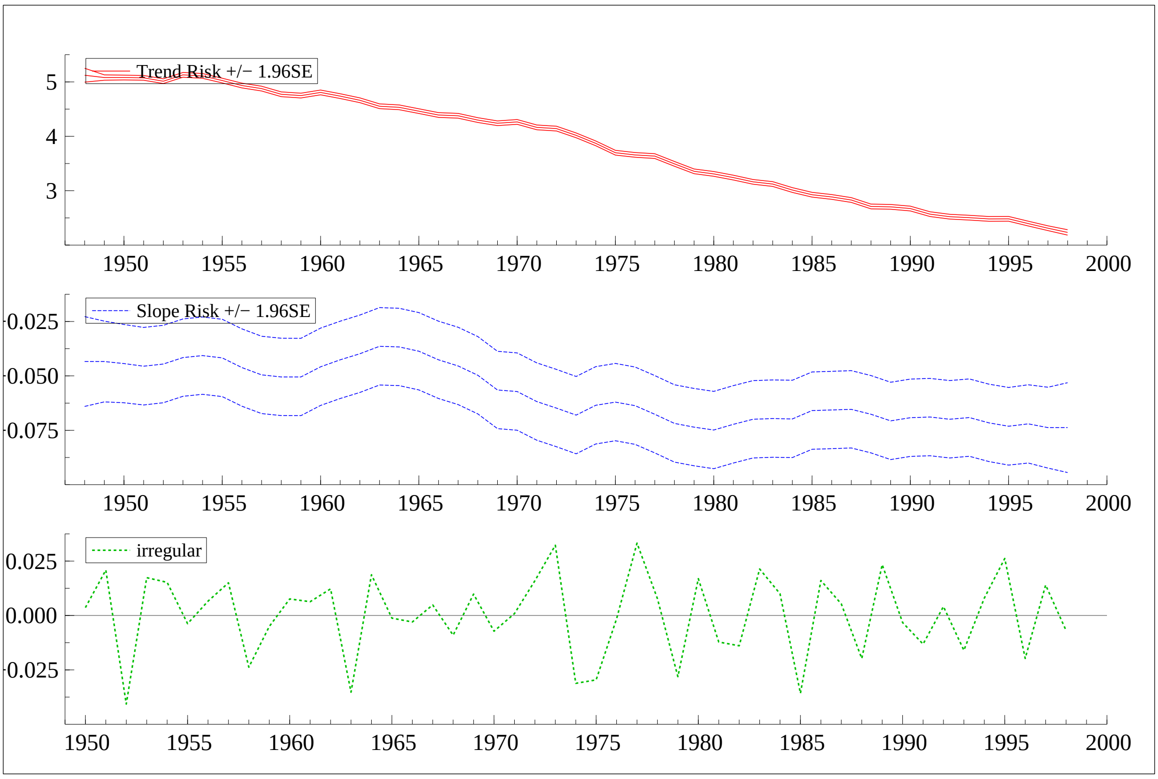

Figure 2. *Dutch development in fatality risk in the years 1948-1998: trend (top) and slope (middle) component. Irregular component for Dutch fatalities (bottom).*

The top graph in *Figure 2* shows the modelled development or trend of the fatality risk $\mu_t^{(2)}$ (corresponding to the third equation in (4)), including its 95% confidence limits. The middle graph displays the development of the slope component $\nu_t^{(2)}$ of the fatality risk (the fourth and last equation in (4)), including its 95% confidence interval. The bottom graph contains the observation disturbances or residuals $\varepsilon_t^{(2)}$ corresponding to the second equation in (3).
Since the value of the slope component in *Figure 2* is negative throughout the observed period, the fatality risk decreased all the time, albeit at varying rates. On the whole, the rate of decrease in fatality risk was smaller at the beginning than at the end of the observed period.

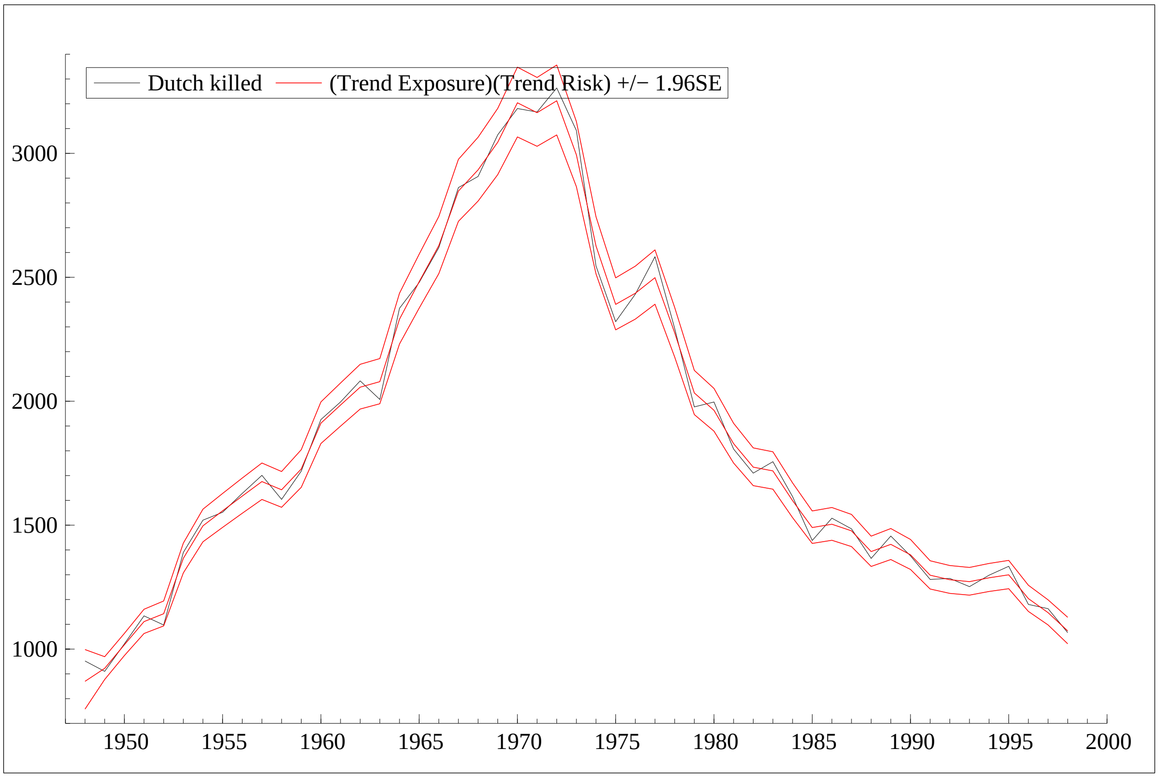

Figure 3. *Dutch development in traffic safety in the years 1948-1998*.

Figure 3, finally, literally illustrates formula (7), and contains both the observed Dutch fatalities as well as the modelled development in Dutch traffic safety, including its 95% confidence interval. The modelled development is the product of the exponent of the trend in exposure (as shown in *Figure 1*) and the exponent of the trend in risk (as shown in *Figure 2*):

$$\hat{F}_t = \exp(\mu_t^{(1)})\exp(\mu_t^{(2)}) = \text{(trend Exposure)(trend Risk)},$$

for t = 1948, …, 1998.

Because the estimated slope disturbance variances are quite small, the analysis was repeated with zero slope disturbance variances. This yields a log-likehood of 139.70, and the value of the AIC for this model equals -2.5040. Since the latter value is larger than the one obtained with the previous model, the stochastic treatment of the slope components gives better results.

For purposes of comparison, the analysis was also performed with deterministic model (8) (i.e., the model where all disturbances in (4) are fixed on zero). This yields a log-likelihood of 22.20, with a value of -0.2587 for the AIC. Since the latter value is much larger than the value of -2.9370 for the stochastic model, the stochastic model fits the data much better than the bivariate classical linear regression model (8). That the deterministic model does not capture the dynamics of the data at all also becomes clear by inspecting the results displayed in *Figure 4*.

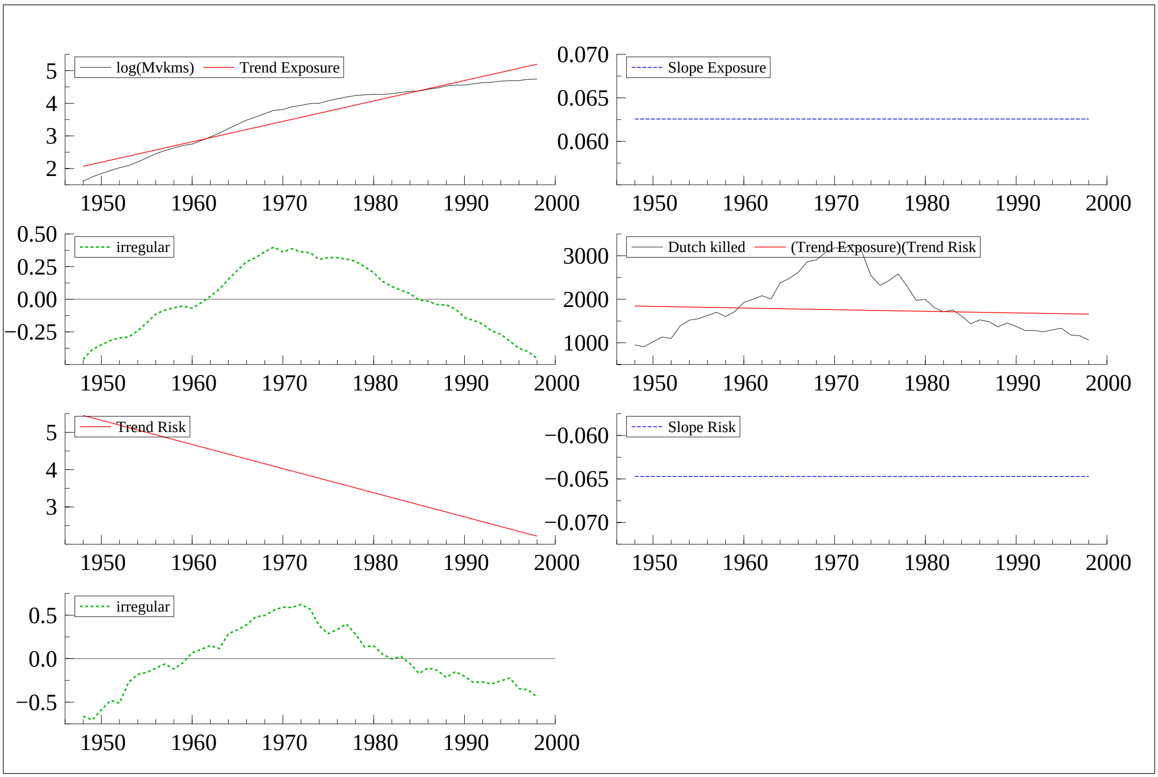

Figure 4. *Results of deterministic treatment of levels and slopes in the basic model.*

The model predictions for the mobility figures in the top left graph of *Figure 4* deviate considerably from the observed mobility figures, and the same applies for the predicted and observed fatality figures shown in the right graph in the second row of *Figure 4*.
Moreover, the residuals for the mobility figures and for the fatality figures shown in the left graph of the second row and in the bottom graph of *Figure 4*, respectively, do not satisfy the assumption of independence at all. In both cases, the observed values are systematically overestimated in the first years of the series, consistently underestimated in the middle years of the series, and again systematically overestimated at the end of the series. Thus, any statistical test or confidence interval based on these residuals will be seriously flawed.
The residuals for the stochastic model shown at the bottom of *Figures 1* and *2*, on the other hand, do satisfy the assumption of independence and can therefore be used to set up reliable statistical tests and confidence intervals.

Forecasts
Since the actual Dutch annual figures for mobility and traffic fatalities are known for the years 1999-2003, the forecasts obtained with the basic evaluation model for these years can be compared with the actual figures. These forecasts are shown in *Figures 5* to *7*, together with the 95% confidence limits of the forecasts.

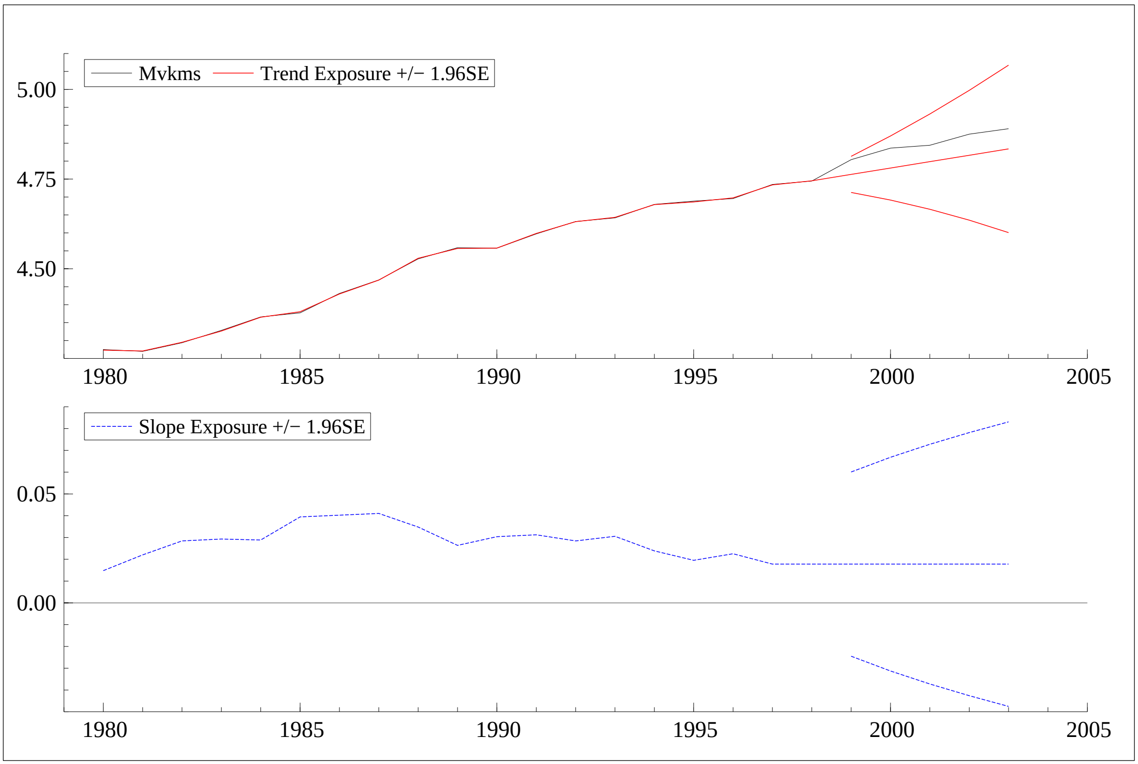

Figure 5. *Forecasts trend and slope exposure 1999-2003 with 95% confidence limits, and log(observed mobility).*

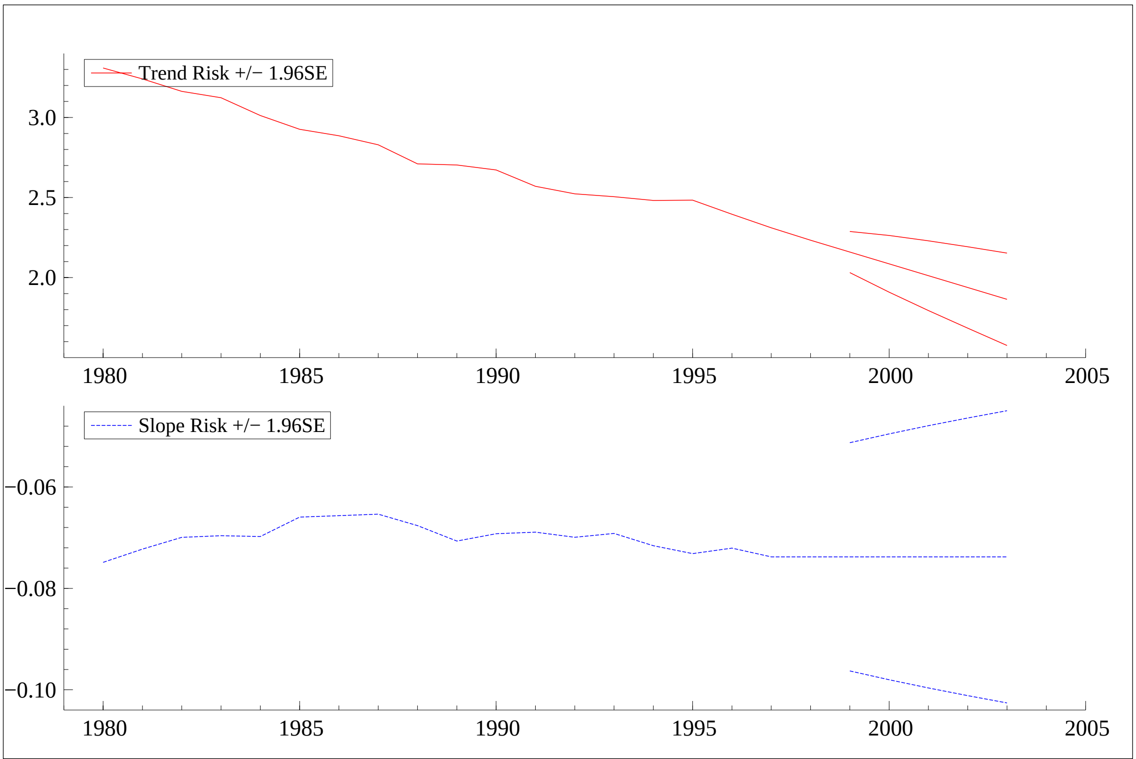

Figure 6. *Forecasts trend and slope fatality risk 1999-2003 with 95% confidence limits.*

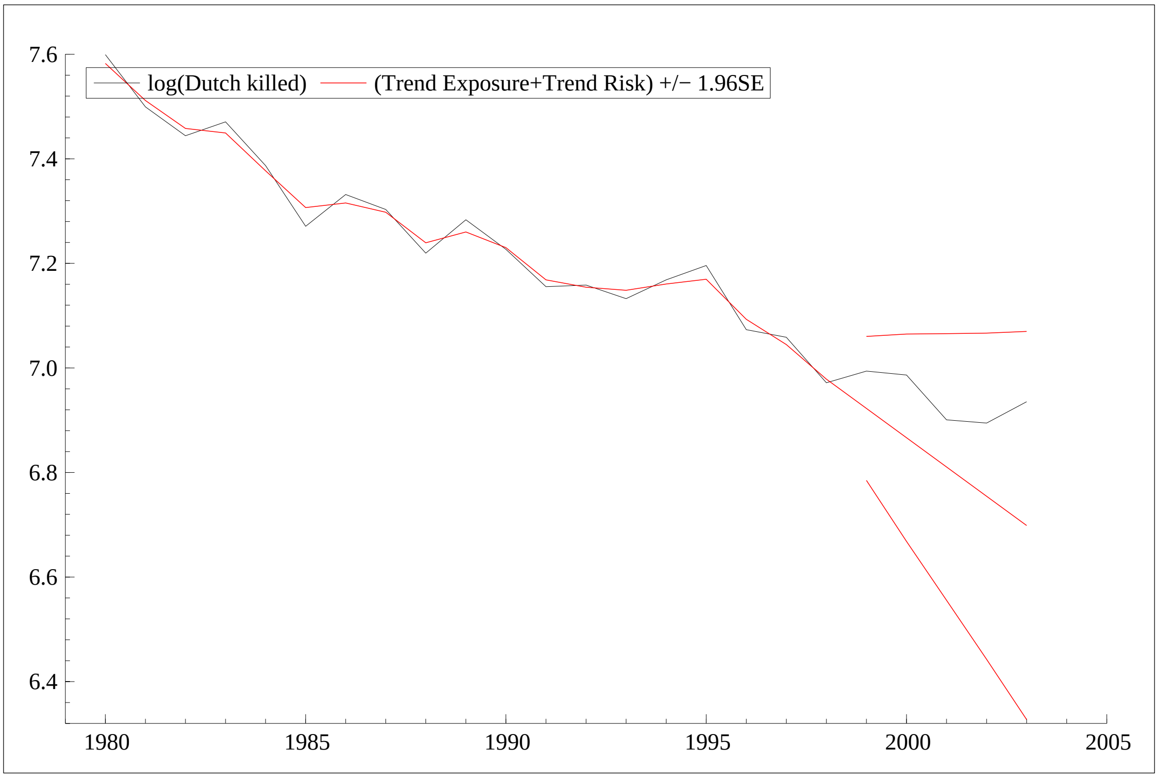

Figure 7. *Forecasts fatalities 1999-2003 with 95% confidence limits, and log(observed fatalities).*

Clearly, during the past five years Dutch mobility has increased more than the forecasts of the model would suggest (see *Figure 5*), and this is also reflected in an underestimation of the actual number of Dutch traffic fatalities (see *Figure 7*).

4. Conclusions and extensions

As we mentioned in the introduction, a central problem in the analysis of repeated measurements over time is that the resulting observations are not independent of one another. Ignoring these dependencies, and applying standard techniques in the analysis of time series therefore yields serially correlated residuals that are unfit for the construction of confidence limits and of statistical tests.
This is clearly confirmed in the illustration presented in *Chapter 3*, where the residuals of a bivariate linear regression model applied to annual Dutch road traffic fatality and mobility data completely fail to satisfy the assumptions of independence.
In Commandeur and Koornstra (2001), the results are discussed of using non-linear regression models in the analysis of traffic safety data. Although the latter models yield a somewhat better fit to the observed fatality and mobility figures than the linear regression model discussed in the present paper, the model residuals are still seriously serially correlated (see *Figures 2* through *6* in Commandeur and Koornstra (2001, pp. 13-17)).
As shown in *Chapter 3* of the present paper, the analysis of these same data with a structural time series model, on the other hand, not only results in a much better fit than the linear and non-linear regression models, but, at least as importantly, in residuals that do satisfy the model assumptions.
Attractive features of the basic evaluation model, moreover, are that
1. the mobility figures are treated as being subject to measurement error, from which exposure is derived as a latent variable.
2. the risk is completely treated as an unobserved, latent variable (which indeed it is).
3. developments in traffic safety, exposure and risk can be simultaneously analysed with one and the same model.

Finally, the basic evaluation model presented in *Chapter 2* is very flexible and can be extended in a number of ways:
1. If quarterly, monthly, weekly, etc. data are available, seasonal components can be added to models (3) and (4).
2. The so-called auxiliary residuals of the basic evaluation model can be inspected to identify structural breaks and outlier observations in the modelled developments.
3. Explanatory and intervention variables can be added to the trend and the slope of the exposure in (4), as well as to the trend and the slope of the risk in (4), as well as to the two observation equations in (3).
4. The basic evaluation model can be used to obtain forecasts for future developments, including their confidence limits (see *Chapter 3* for an example).
5. Last but not least, the basic evaluation model can be extended to the simultaneous multivariate analysis of disaggregated developments in road safety.

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