THE USE OF MULTIPLICATIVE MODELS FOR ANALYSIS OF ROAD SAFETY DATA

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SUMMARY

Accident ratios are analysed with regard to the variables road-surface skidding resistance and hourly traffic volume. In a first analysis the Additive Conjoint Measurement model (ACM) is used to investigate to what extent the accident ratios can be described as a result of independent contributions of skidding resistance and traffic volume. Furthermore is considered whether these contributions have to be combined in an additive or multiplicative way. Based on the results of this investigation a second analysis took place in which a stochastic interpretation of the data is combined with the multiplicative model. This Weighted Poisson model (WPM) is in fact a generalisation of the log-linear model, recently proposed for the analysis of contingency tables.
It has been concluded that the multiplicative model describes the data better than the additive model. Moreover that there is no interaction between skidding resistance and traffic volume in their effect on accident ratios. The pictures of the relation between accident ratios and both variables are shown and the statistics regarding the contributions of the variables.

INTRODUCTION

In 1966, the Institute for Road Safety Research SWOV in The Netherlands set up a Working group on Tyres, Road Surfaces and Skidding Accidents. The terms of reference of Sub-committee V of this Workking group were to establish the number of skidding accidents.
It was also to consider the part played by road-surface skidding resistance in accident occurrence. The following organisations were represented on the Sub-committee: the State Road Laboratory RWL, Delft, the Traffic and Transportation Engineering Division Rijkswaterstaat DVK, The Hague, and the Institute for Road Safety Research SWOV, Voorburg.
In order to investigate the extent of the skidding problem, accidents occurring on dry road surfaces were compared with those on wet surfaces with and without rainfall. The role of skidding resistance was investigated only as regards accidents during rainfall. In this latter investigation a number of variables such as speed and visibility were disregarded for practical reasons. The investigations did, however, cover hourly traffic volume, traffic performance, type of road and type of vehicle.

This contribution is not a report on the research. This is given in Schlösser (1977).
The present intention is only to describe the methods of analysis employed and the consequent conclusions regarding the relationship between accidents, traffic performance, hourly traffic volume and road-surface skidding resistance. The analysis in based on the assumption that traffic can play a part in accident occurrence in two ways. On the one hand, if there is more traffic the expected number of accidents will increase owing to the larger number of accident-suspectible road users; in other words, exposure increases. As to this point, the number of accidents is likely to increase proportionately to traffic performance. On the other hand, at higher traffic volumes the accident hazard will increase for every individual road user; i.e. accident-susceptibility increases.
The analysis was adjusted for the extent to which exposure plays

a part. For this purpose, accidents occurring in a given time on a given road section were divided by the number of vehicle-kilometers driven during that time on that road section. These accident ratios are analysed. Besides the adjustment for vehicle-kilometers, the hourly traffic volume is used to explain the difference in accident ratios, in order to ascertain the influence of traffic volume on accident-susceptibility. An endeavour is therefore made to define the accident ratio as a function both of road-surface skidding resistance and hourly traffic volume. It is reasonable to assume that the increase in accident-susceptibility will not be the same on all roads. Consequently, roads are divided into two types. Type I comprises motorways: roads with split-level junctions and separate carriageways each with at least two lanes and generally one shoulder. Type II comprises other primary national highways, mainly single-carriageway roads with two lanes, level junctions and sometimes slow traffic.

1. DATA

The accident data required for the research were obtained from Rijkswaterstaat (Department of Roads and Waterways).
The locations, times and dates of the accidents were recorded, and whether it was raining.
For road type I the traffic volumes were divided into 20 classes with a width of 100 vehicles per hour for each direction; for type II into 15 classes with a width of 200 vehicles per hour in both directions. The coefficient of longitudinal force for a wet surface was determined for each road section. The coefficients were divided into 9 skidding-resistance classes with a width of 0.05 units of measurement from $\leqslant$ 0.36 to $>$ 0.71.
From the location, data and time, the appropriate skidding-resistance and hourly volume classes were determined for each accident. Since the highest resistance class also includes accidents on wet surfaces during dry weather, it was completely eliminated from the investigation.
From the length of the road, the distribution of hourly traffic volumes and the duration of rainfall, the number of vehicle-kilometers was calculated for each combination of skidding resistance and traffic volume, separately for workdays and weekends and adjusted for month and year.
Next, the accident ratio was determined for each resistance-volume combination by dividing the number of accidents by the relevant number of vehicle-kilometers.
This resulted in two tables of accident ratios corresponding to the two types of roads.

2. ANALYSIS

The intention of the analysis is to examine how the accident ratio (A) depends on hourly traffic volume (T) and road-surface skidding resistance (R).

The obvious approach to such problem is to apply multiple linear regression (MLR) to the data. This would result in the following description of the expected value of the dependent variable as a linear combination of the independent variables:

$$E(A) = aR + bT + c \qquad (1)$$

If a, b and c are known, the value of E(A) can be found by filling in the values of R and T.

With MLR, those values of a, b and c are sought which predict the A values as closely as possible from the R and T values. But if we examine the assumptions for this regression model more closely, the straight foreward application to the given data leads to a number of difficulties.

Assumption 1: the assumption of linearity

This assumption means that if the independent variable R is kept constant, the dependent variable A is linearly related to the variable T van vice versa. In such a case we speak of an MLR model which is linear in the independent variables. This assumption of linearity entails a number of problems. Firstly, the manner in which the resistance classes are established will determine how the accident ratio is related to skidding resistance. There is no prior reason for assuming that this relationship will be linear. Moreover, the relationship is less clear as regards traffic volume. It is indeed not unreasonable to say that the accident ratio increases within certain limits with traffic volume. But it is possible that it increases at a very low volume, while it decreases at a high volume and low capacity. Overal inspection of the data suggests that this assumption is correct. Moreover the question remains whether the relation is linear in the central range.

In order to meet these problems, it is possible to extent the MLR model with terms that are quadratic in the independent variables or with terms of a still higher order.

Assumption 2: the assumption of additivity

According to this assumption, the dependent variable can be written as a (weighted) sum of independent variables. As regards assumption 2, the following can be said:

Suppose the probability of a given accident occurring on a road surface belonging to skidding-resistance class j (j = 1, ..., m) is indicated as $p(R_j)$, and the probability of this accident occurring in traffic-volume class i (i = 1, ..., n) is indicated as $p(T_i)$. Now if we assume that both events R_j and T_i are independent of each other (which means that the probability distribution over the resistance classes is the same for every volume class and vice versa), it follows that the probability of an accident for the combination of traffic-volume class i and skidding-resistance class j can be written as the product of the (marginal) probabilities $p(T_i)$ and $p(R_j)$, viz:

$$p(T_i \cap R_j) = p(T_i) \cdot p(R_j) \qquad (2)$$

This consideration should lead to the choice of a multiplicative model instead of an additive model.

We would verify this hypothesis by extending the MLR model aready mentioned (1) by adding an R.T term, i.e.:

$$E(A) = aR + bT + cR.T + d \qquad (3)$$

If for each class of T the relation between A and R is linear (and vice versa), and the hypothesis of multiplicity is true, then (3) reduces to $E(A) = cR.T + d$.

A following suggestion could then be not to analyse the data themselves, but to make the analysis with the logarithm of the data.

In general, if $Z = XY$, then $\log(Z) = \log(X) + \log(Y)$ and multiplicity changes into additivity. From this analysis the required information could then be derived regarding the contribution of T and R to A. From formula (2) and the discussion of the linearity assumption, however, the suggestion obtrudes to include a separate parameter in the model for each class of R and T. Within the MLR model this is possible, for example by using an $m-1^{st}$ degree polynomial in R and an $n-1^{st}$ degree polynomial in T.

A model in which this requirement is met in a slightly different way is the Additive Conjoint Measurement (ACM) model.

3. ADDITIVE CONJOINT MEASUREMENT

3.1. General description

We speak of an ACM model if an order relation has been assumed between the dependent variable and the sum of arbitrary functions of the independent variables.
The ACM model has the requirement of additivity, but here too multiplicity can be converted into additivity by using a logarithmic transformation.
In general, let X and Y be the independent variables and Z the dependent variable. Then the linearity requirement is replaced by the requirement of arbitrary functions f on X and g on Y with the aid of which Z (or a logarithmic transformation of Z) can be described as a function of X and Y.
For every Z_{ij} value belonging to the combination (X_i, Y_j):

$$E(Z_{ij}) = f_i + g_j + c \qquad (4)$$

Where $f_i = f(X_i)$, $g_j = g(Y_j)$ and c is a general parameter.

If the n times m Z values are regarded as a vector Z and the n + m + 1 parameters as a vector θ, then model (4) can be written $E(Z) = V\theta$, in which V is then called the design matrix. V is then a matrix of ones and noughts in such a way that each Z-value has parameters added to it in conformity with the indices i and j.
In analysis of variance the emphasis in the specification of V is verification of hypothesis according to some experimental design (hypothesis testing), in ACM it is more a matter of combined measurement of the variables X and Y with the aid of the parameters (parameter estimation). With the MLR models mentioned, the matrix V would be replaced by a matrix whose column vectors are the values of the independent variables or their polynomials. Interaction terms could be added to these MLR models, such as XY, X^2Y and so on. In ACM analysis, these effects are assumed to be non-existent.

So far, two alternatives have been mentioned for analysis.
The first possibility is to apply the analysis directly to accident ratios; the second is to apply it to their logarithms. Here we assume that the order relation between the dependent variable and the independent variables is linear or logarithmic.

Another possibility, following a method developed by Kruskal (1965), is to make an analysis seeking for the monotone non-descending transformation of Z which, if filled in for Z, gives a solution of equation (4). In this case we speak of an ACM model. If it is subsequently examined which monotone transformation leads to the best fit of the ACM model, the above arguments regarding additivity or multiplicity can still be verified. For example, if the monotone transformation is a linear transformation, then the ACM solution is identical to the solution that results from application of the common linear model as in analysis of variance. If it is a logarithmic transformation then a multiplicative model has been applied to the data.
Nelder & Wedderburn (1972) used fixed transformation functions to choose between an additive and other models. They speak of general linear models if an other transformation then the identity transformation is used. This results from the fact that after the transformation the model is linear.
Here we use no fixed transformation of Z but seek for the best monotone transformation to describe the data with model (4). If we generalize MLR models in this way, one mostly speaks of non-metric MLR analysis, because it is, in fact, assumed that only ordinal information exists concerning Z.
Formulated somewhat more exactly, Kruskal's method amounts to the following: Suppose f and g are known, then for each Z_k and Z_l there is a

$Z_k^* = f(X_k) + g(Y_k)$ and a $Z_l^* = f(X_l) + g(Y_l)$, for which,

if $Z_k \geqslant Z_l$, then $Z_k^* \geqslant Z_l^*$ too.

Or:

$$Z_k^* = f(X_k) + g(Y_k) \geqslant Z_1^* = f(X_1) + g(Y_1) \qquad (5)$$

if, and only if $Z_k \geqslant Z_1$

in which k and l are indices continuing through the resistance-volume combinations (1,1), ..., (1,n), ..., (m,n).
If f and g are unknown, then we have to find a monotone transformation Z^* of Z and values for f and g such that (5) holds.

As a rule, a transformation of Z will be possible only up to a certain level. An effort will thus have to be made to find the transformation for which the model gives the best possible description of the data. As a criterion for optimum description, a least-squares criterion is chosen as used in MLR. In other words, let Z_k^+ be the value belonging to a given monotone transformation of Z_k. Given the values of Z^+, we can look for a solution of f and g. From the values of f and g we can compute the values of Z^*. Finally the monotone non-descending transformation is sought for which the sum of the discrepancies (S) between the Z_k^+ and Z_k^* values is as small as possible. Or, more precisely, for which:

$$S = \min_{Z^+} \min_{\theta} \left[\sum_k (Z_k^+ - Z_k^*)^2 \Big/ \sum_k (Z_k^* - \bar{Z}^*)^2 \right]$$

The denominator in this expression is merely a scale factor. In an interative process seeking simultaneously the best fitting functions f and g and monotone transformation of Z, the Z values themselves are chosen as the starting configuration.
By comparing the value of S found with the starting configuration S_d, with S_{mon} of the monotone transformation, it is possible to examine how far the solution can be improved if we allow a monotone transformation for Z. If we also apply the analysis to the log-Z values, we again obtain a starting solution with matching S_{log} which, compared with S_d, shows whether it is better to speak of an additive

or multiplicative model, while S_{log} compared with S_{mon} (identical of course for both starting situations) again shows how this solution can be improved.
If the hypothesis concerning multiplicity is correct, we expect
$S_d > S_{log} = S_{mon}$

The testing of hypotheses regarding monotone transformation is not easy. To get an indication of the significance of the results a Monte-Carlo study is made. This results, under the assumption of normal distributed S-values, in the testing of the hypotheses by means of t-statistics.

3.2. Results**

Table 1A gives the accidents for road type I and Table 1B the relevant vehicle-kilometres. Tables 2A and 2B give the same values for road type II.
Figure 1 gives the solution for the eight values of function f in formula

$$E(A^*_{ij}) = f(R_i) + g(T_j)$$

which is a specification of (5) with regard to the given accident data.
In this formula R_i is the skidding-resistance class i, T_j the traffic-volume class j and A^*_{ij} the relevant accident ratio after monotone transformation. For classes 2 to 7, the size of the parameters decreases more or less linearly with the class value.
If the multiplicative model is correct (i.e. if the monotone transformation is found to be logarithmic) and $f(R_i)$ is indeed linear, this means that the relationship between accident ratio and resistance class is exponential.

** The computer programme ACM, written in PLI by J. de Leeuw, Leyden State University, was used for analysing the data.

Figure 2 gives the solution for the values of function g. The relationship is not so clearly interpretable with road type I. It can, however, be inferred that the accident ratio increases with traffic volume, except at the ends of the scale. At very low volumes the accident hazard increases, and at very high volumes it decreases. For road type II we do not see these peripheral effects.

Figures 3 and 4 show the transformations of the accident ratios for both types of road. For road type I it follows from Figure 3 that the transformation can indeed be regarded as a log-transformation. For road type II this is not the case, as can be seen in Figure 4. It will be found later that the extra curvature for road type II does not contribute much to improving the solution, as related to a log-transformation.

In order to examine this more closely, the S values of each of the fit procedures are important. These are given in Table 3, for the least-squares solution of the original data, the log-data and the ultimate solution after monotone transformation respectively. The table shows that the stress after log-transformation over the data becomes smaller, while it is of course higher than that of the solution after monotone transformation.

In order to obtain an idea of the degree to which the established differences in stress are significant, a Monte Carlo study was made. The procedure is as follows:
Allot the established accident ratios at random to the R and T classes and apply an ACM analysis to these data and the relevant log-data. Repeat this very many times (for economy, this was done only forty times). Calculate the means and standard deviations.
The values in Table 3 can be compared with these means. The Monte Carlo data are given in Table 4. It follows from this tables (on the assumption that the stresses are distributed normally):
A. That the fit of the original data and log-data is very significantly better than random. For the log-data, for instance, we find a t-value of

$$t = \frac{.1334 - .6928}{.058} = -9.64 \text{ (df = 39)}$$

B. That the stress values for analysis with the Monte Carlo data and Monte Carlo log-data do not differ, as was to be expected (t = .183); the difference between the stress values S_d and S_{log} in the original analysis is .0515. This difference, though fairly high (t = .0515/.038 = 1.35), is not significant. For road type II, the absolute difference is greater. It is thus reasonable to choose the multiplicative model.

C. That the mean (trivial) reduction in stress after monotone transformation of data for the Monte Carlo data is .0603. For the original analysis this value is .0180 for road type I and .0441 for road type II, and there is thus no reason to assume that monotone transformation produces an additional improvement which is not trivial. This conclusion strengthens the view that the multiplicative model is correct.
It also shows that the bending in the curve in Figure 4 already mentioned hardly improves the solution.

4. STOCHASTIC INTERPRETATION OF THE MULTIPLICATIVE MODEL

Assuming that the occurrence of accidents can be described as a Poisson process with parameter λ and that the accidents are multinomially distributed over het skidding-resistance and traffic-volume classes while the volume and resistance variables have a mutually independent influence on the probability of an accident, then:

1. For each resistance class R_i with multinomial probability p_i, and each volume class T_j with multinomial probability q_i, accidents can be described according to a Poisson distribution with parameters λp_i and λq_j.

2. For each cell (R_i, T_j) the accident frequencies are Poisson distributed with parameter $\mu_{ij} = \lambda . p_i . q_j$.

4.1. Log-linear models

In recent years methods of analysis have been developed specially for data collected in the form of contingency tables. The subdivision of the data into volume and resistance classes described above is an example of such a contingency table. If it can in fact be assumed for the values in the cells of the contingency tables that they are Poisson distributed or multinomially distributed, these methods can be employed. Within the Poisson models one tries to describe the Poisson parameters, which may differ from cell to cell, in terms of the variables that constitute the contingency table. The multiplicative model described above is a specific example of this. The Poisson parameter for each cell is described there as being composed of three part-parameters: a general parameter (identical for each cell) λ, one (identical for each cell is one row of the contingency table) p_i, and one (identical for each cell in one column) q_j. In other words, restrictions are imposed on the ultimate Poisson parameter of every cell relating to the position in the row and column of such cell in the contingency table. However, it is a single choice from a number of possible restrictions. If we say,

for instance, that road-surface skidding resistance has no influence at all on accidents, viz. that all p_i's are the same, the model could be simplified. For each cell, its Poisson parameter would then be equal to λq_j (one general part-parameter and one part-parameter for the location of the cell in a column).
The most general form in which the parameters can be written is:

$$\mu_{ij} = \lambda . p_i . q_j . r_{ij}$$

or, if we take the logarithm:

$$m_{ij} \; (= \log \mu_{ij}) = \alpha + \beta_i + \gamma_j + \delta_{ij} \qquad (6)$$

in which the terms after the = sign indicate the logarithms of the previous expression.
Models which try to give such a representation of the Poisson parameters μ_{ij} are therefore known as log-linear models. A detailed description is given in Goodman (1970), Haberman (1974) and Bishop, Fienberg & Holland (1975).
The multiplicative ACM model is in fact also a log-linear model, but without this stochastic interpretation. The multiplicative model comparable with model (4) imposes the additional restriction that $\delta_{ij} = 0$, for all combinations (i, j). The data in a contingency table can always be constructed perfectly with the aid of (the saturated) model (6).
It can now be checked, for instance, whether the (non-saturated) multiplicative model $m_{ij} = \alpha + \beta_i + \gamma_j$ represents the data significantly worse than model (6).
An example of applying such a type of analysis to road-traffic problems (but with a differing model description) is found in Rash (1973).

4.2. Weighted Poisson models

The application of log-linear models to contingency tables in which accidents are given seems warranted: the assumption that the numbers of accidents is Poisson distributed, is generally accepted. If we are dealing with accidents ratios instead of accidents such an analysis is not directly applicable.

De Leeuw (1975) describes a more general model applicable to Poisson distributed variables corrected by dividing the variables by a constant. In other words: Poisson distributed variables are first weighted before being analysed. The accident ratios can be regarded as such weighted variables. A drawback to this is that strictly speaking vehicle-kilometres are not correcting constants but in fact stochastic variables. The variance of these variables, however, is many times smaller than that of the accident variables, and the drawback will not be very important in practice. A second drawback applying to all log-linear analyses is that the model is only verifiable asymtotically, which means that it holds good in so far as the expected number of accidents is large enough for each cell. In the present case, this condition certainly does not apply to each cell, which makes the model difficult to test. A detailed description and an example of using weighted Poisson models can be found in De Leeuw & Oppe (1976).

The same type of model has been used in Andersen (1977). There the model is applied to lung cancer cases in different Danish cities for different age-groups weighted corresponding to the number of inhabitants.

4.3. Results**

The initial analysis for road type I is applied to the data of skidding-resistance classes 2 to 7 and traffic volume classes 1 to 16, while in addition the sum of classes 17 to 20 was included as a class. The results are shown in Figures 5 and 6 together with those of the ACM - analysis for the log-data.

In a second analysis, each two volume classes were combined, and eight volume classes and the remaining class were therefore ex-

** The computer programma WPM written the PLI by the author was used for analysing the data amined. The result of this analysis is also shown in Figure 5 as far as regards the parameter estimates for the volume classes. On the whole, there is close agreement between the ACM-log solution and the WPM solution. The solution of the doubled volume classes shows that the instability of the curves has been greatly reduced, and hence the relation between accident-susceptibility and volume class is more clear. The information in Table 5 shows from the size of the Chi-squared value that especially road-surface skidding resistance determines the difference in accident-susceptibility (X^2 = 373.40, df = 5).

But the difference in traffic volume also makes a substantial contribution (X^2 = 72.32, df = 16).

There is no significant interaction (X^2 = 83.51, df = 80) in the first analysis; on the other hand there is a moderately significant interaction if the volume classes are combined (X^2 = 56.48, df = 40). All this greatly favours acceptance of the multiplicative model as such and to a lesser extent the omission of the interaction term.

The latter means in fact that the relation between accident ratios and road-surface skidding resistances is the same for each volume class and that there is only a difference in level between accident ratios for the volume classes. In terms of adopting measures, this means that the same norm can be applied everywhere on type I roads. The effectiveness will, of course, depend on the volume of traffic.

For type II roads, skidding-resistance classes 1 to 7 were analysed. For the traffic-volume classes, the values of classes 1 to 10 were used and those of the 11th to 15th classes were added. The results are shown in Figures 7 and 8. Here again, there is close agreement between the ACM-log and WPM solutions. Table 5 shows that the main contribution to the difference in accident-susceptibility is made by skidding resistance (X^2 = 331.41, df = 6) and that traffic volume also makes a very significant contribution (X^2 = 120.72, df = 10). Within the multiplicative model, however, there is a very significant interaction for road type II (X^2 = 191.89, df = 60), and it can therefore be said that the multiplicative model without the

interaction term does not fit as well here as for road type I.
In a second analysis, resistance classes 1 and 2 and 6 and 7 were combined, while volume classes 9 and 10 were put in the remaining class. Hence, the number of cells with few observations was greatly reduced. Here again, interaction was found to be significant (X^2 = 142.27, df = 32) and it is not reasonable to assume that the interaction can be explained by the number of observations within cells being too small.
An explanation might be the great diversity of type II roads as mentioned in the introductory remarks and the fact that carriageways of this type are not usually separated. Moreover, accidents at junctions may distort the picture. In this case the use of vehicle-kilometres is certainly not the best correction for exposure.

To sum up, the conclusions are:

1. The ACM model gives a good description of the log-data.
2. The application of the WPM model for road type I data, based on the results of the ACM analysis, supports the hypothesis that road-surface skidding resistance and traffic volume have independent effects upon accident occurrence.
3. Consequently a description of accident ratios can be given in terms of only one of the two variables. The practical implications of this as regards measures to be adopted have been worked out by Schlösser (1977).

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T \ R	2	3	4	5	6	7	8
01		3.50	5.00	34.50	52.00	7.50	
02	3.00	1.00	6.50	23.75	33.25	8.75	.25
03	2.00		5.50	38.75	55.00	16.00	1.25
04	4.00	4.50	8.50	52.00	56.00	21.50	
05	3.00	2.00	6.25	58.75	67.50	11.50	
06	3.50	1.50	6.50	47.75	52.00	11.75	
07	15.00	2.50	8.50	52.50	81.25	10.25	
08	13.50	3.75	15.25	61.00	77.25	15.75	
09	8.00	2.50	13.25	81.50	66.50	7.75	
10	4.50	5.25	14.00	70.75	66.25	9.75	
11	4.00	3.75	18.50	85.75	85.50	7.00	1.00
12	2.50	1.50	26.25	66.75	58.25	7.25	1.00
13		.25	23.00	54.50	43.25	8,50	
14	3.50	1.50	11.75	59.50	34.50	7.25	
15		1.50	10,50	44.25	23.25	2.00	
16		.75	12.75	35.25	21.25	1.00	
17		.75	6.25	25.75	5.75		
18			6.75	27.75	3.00		
19			2.25	12.25	6.50		
20		.50	10.25	48.50	11.25	1.00	

Table 1A. Distribution of number of accidents on road type I according to road-surface skidding resistance (R) and traffic volume (T).

The fractions are the result of dividing accidents into classes where the class cannot be indicated precisely.

T \ R	2	3	4	5	6	7	8
01	79	140	488	4145	9598	4204	610
02	116	165	711	6219	13662	5457	485
03	228	238	1273	11675	25830	11525	877
04	361	400	1699	15613	28172	11457	493
05	193	594	1610	14431	29096	10183	223
06	239	563	1586	14166	31019	8579	195
07	442	626	1539	13952	31060	7508	178
08	492	579	1729	14699	30537	6276	128
09	386	470	1585	14109	25306	4942	100
10	264	475	1518	12865	22231	3524	78
11	239	404	1715	11794	19160	2485	50
12	172	234	1667	10480	15703	2238	32
13	77	153	1029	8085	11093	1483	22
14	78	102	863	6132	8001	926	13
15	40	67	570	4453	5502	551	5
16	6	51	475	3129	3669	442	
17	2	45	495	2362	2782	284	
18		26	379	1702	1798	145	
19		19	236	1513	1454	115	
20	4	26	875	4148	3373	233	

Table 1B. Classification of number of vehicle-kilometres according to road-surface skidding resistance (R) and traffic volume (T) for road type I.

T \ R	1	2	3	4	5	6	7	8
01	8.00	24.00	20.00	49.00	189.00	369.50	93.50	6.00
02	14.00	50.00	57.00	92.50	290.25	487.75	130.00	9.00
03	16.00	35.00	40.00	78.50	323.75	439.25	83.00	1.00
04	11.00	21.00	38.00	70.00	309.50	357.50	43.00	1.00
05	4.00	21.00	33.00	63.50	197.00	168.00	19.00	
06	4.00	12.00	29.00	47.00	163.50	116.00	13.00	
07	1.00	9.00	13.00	36.00	83.00	58.00	6.00	2.00
08		7.00	4.00	13.00	66.00	41.00	3.00	
09	3.00	1.00	6.00	11.00	39.50	31.00	1.00	
10	2.00		1.00	4.00	29.00	17.00	1.00	
11	2.00			4.00	17.50	32.00		
12	2.00	1.00		6.00	13.00	8.00		
13	1.00	1.00		1.00	8.00	3.00		
14					3.00	1.00		
15						4.00		

Table 2A. Classification of number of accidents on road type II according to road-surface skidding resistance (R) and traffic volume (T).

The fractions are the result of dividing accidents into classes where the class cannot be indicated precisely.

T \ R	1	2	3	4	5	6	7	8
01	620	2198	2919	4866	24512	60184	21175	1883
02	860	3470	5521	12544	53617	112993	36351	1583
03	781	2425	3886	11427	54940	86961	20241	583
04	387	1192	2543	7840	45977	55499	9680	491
05	145	589	2157	6800	28100	30695	4691	246
06	58	234	1182	4566	16184	19079	2570	129
07	30	53	436	2392	9722	12643	1676	46
08	38	34	294	1424	5525	7609	945	11
09	15	16	113	708	3070	4724	469	8
10	19	8	57	302	1946	3361	430	6
11	20	11	36	155	1594	2765	299	2
12	18	8	17	125	1109	1706	171	
13	7	3	7	66	337	622	47	
14		4	4	16	83	135	18	
15		2	3	1	182	301	12	

Table 2B. Classification of number of vehicle-kilometres according to road-surface skidding resistance (R) and traffic volume (T) for road type II.

	S_d	S_{log}	S_{mon}
road type I	.1849	.1334	.1154
road type II	.2355	.1230	.0789

Table 3. Stress values for solution of data, log-data and ultimate ACM solution for road types I and II.

	S_d	S_{log}	S_{mon}	$S_d - S_{log}$	$S_{log} - S_{mon}$
mean	.6939	.6928	.6325	.0011	.0603
s.d.	.054	.058	.054	.038	.038

Table 4. Mean stress values for solutions of Monte Carlo data and the relevant standard deviations for data of road type I. Number of data sets 40.

Effect	X^2	DF	$X^2_{.95}$
Road type I, original set:			
T	72.3155	16	26.29
R	373.4048	5	11.07
T x R	83.5095 NS	80	101.88
Road type I, added:			
T	51.80	8	15.51
R	377.33	5	11.07
T x R	56.48	40	55.76
Road type II, original set:			
T	120.72	10	18.31
R	331.41	6	12.59
T x R	191.89	60	79.08
Road type II, added:			
T	215.92	8	15.51
R	652.44	4	9.49
T x R	142.27	32	46.19

Table 5. Results of the four WPM analyses. The source is given under "effect"; against these, the chi-squared values (X^2) with the relevant degrees of freedom (df) and the chi-squared limits belonging to the 5% level ($X^2_{.95}$).

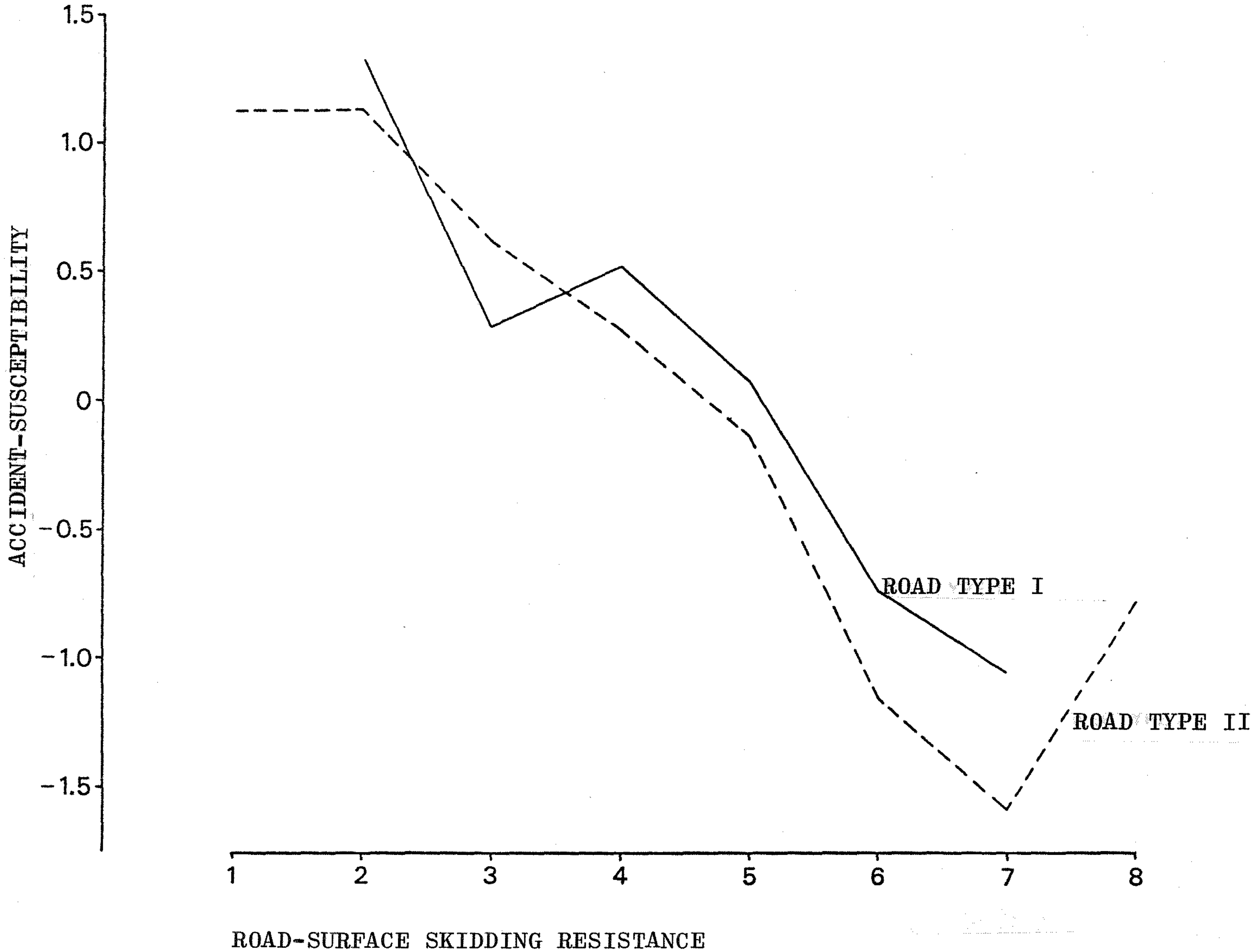

Figure 1. ACM_{mon} solution for skidding-resistance classes of road types I and II.

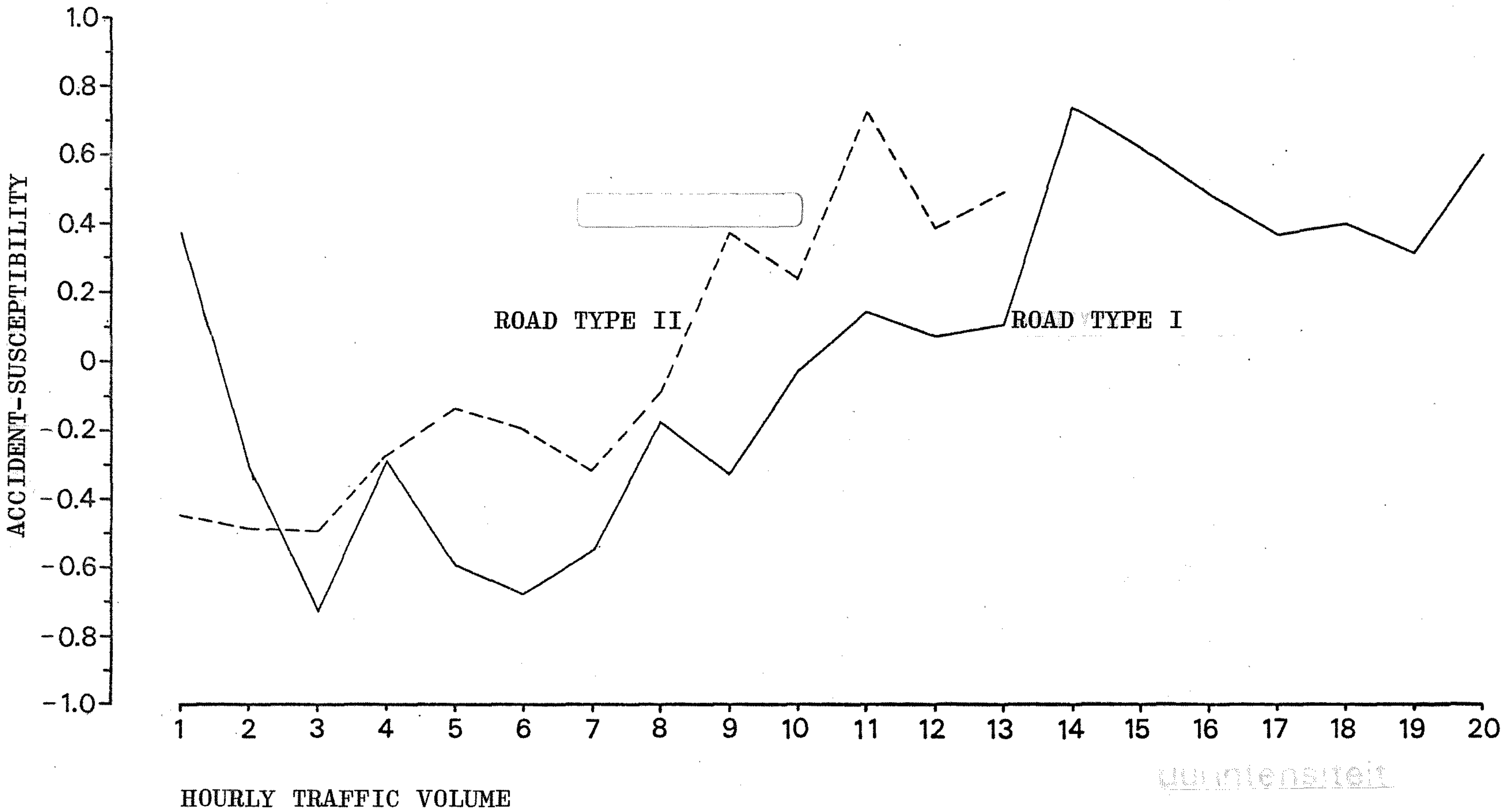

Figure 2. ACM$_{mon}$ solution for traffic-volume classes of road types I and II.

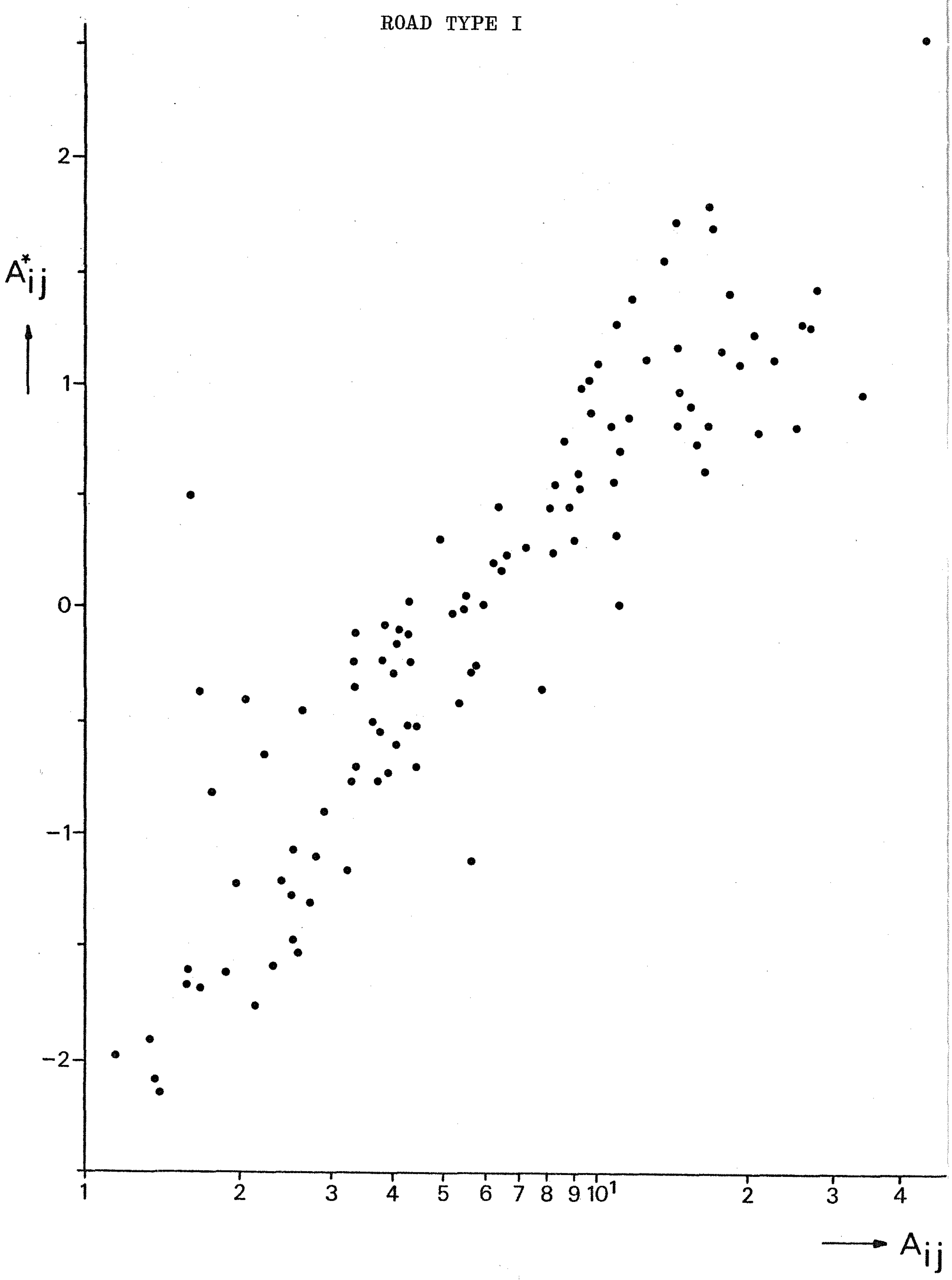

Figure 3. Monotone transformation A^*_{ij} of accident ratio's A_{ij} for data of roadtype I belonging to ACM-analysis.

ROAD TYPE II

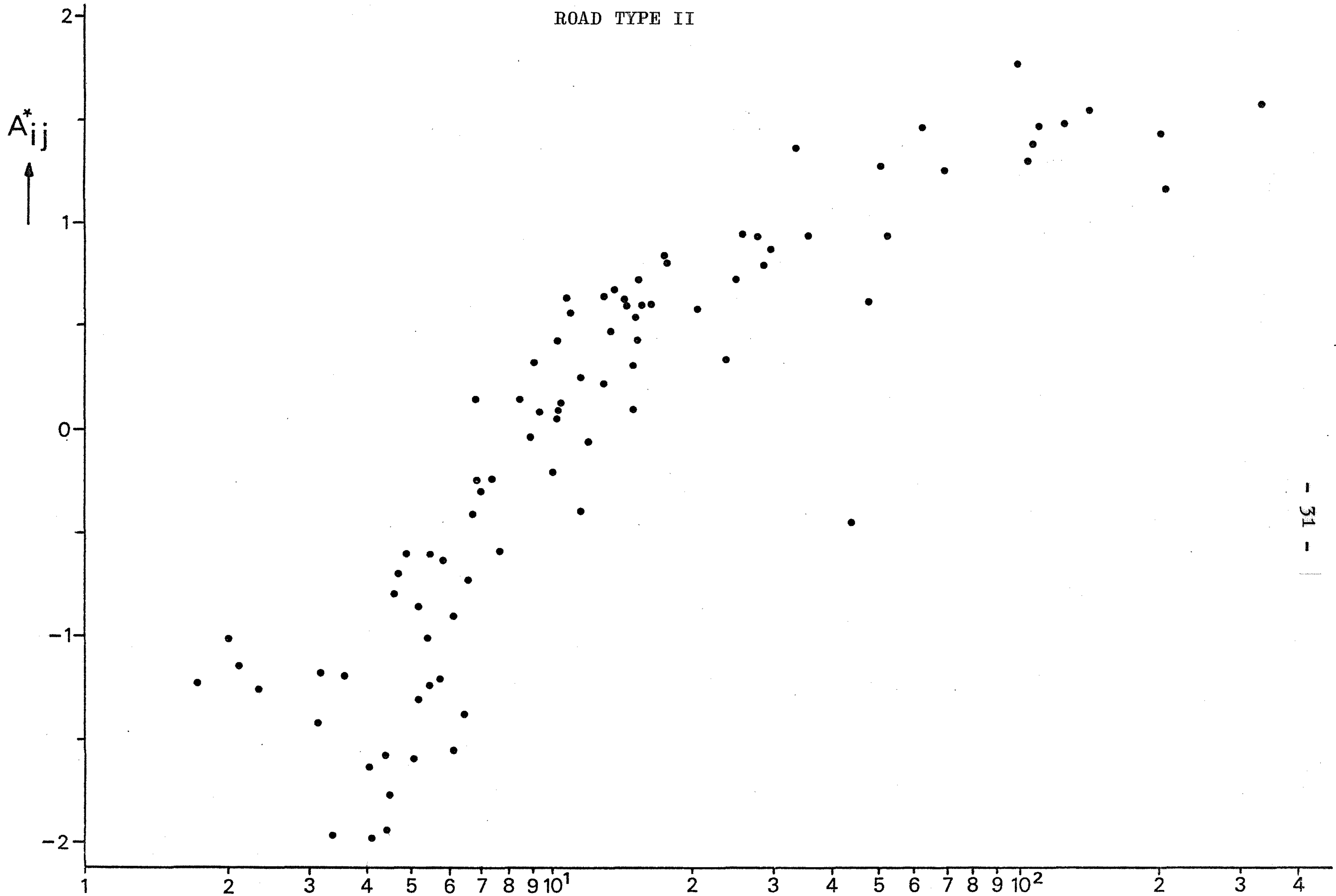

Figure 4. Monotone transformation A^*_{ij} of the accident ratio's A_{ij} for data of road type II belonging to ACM-analysis.

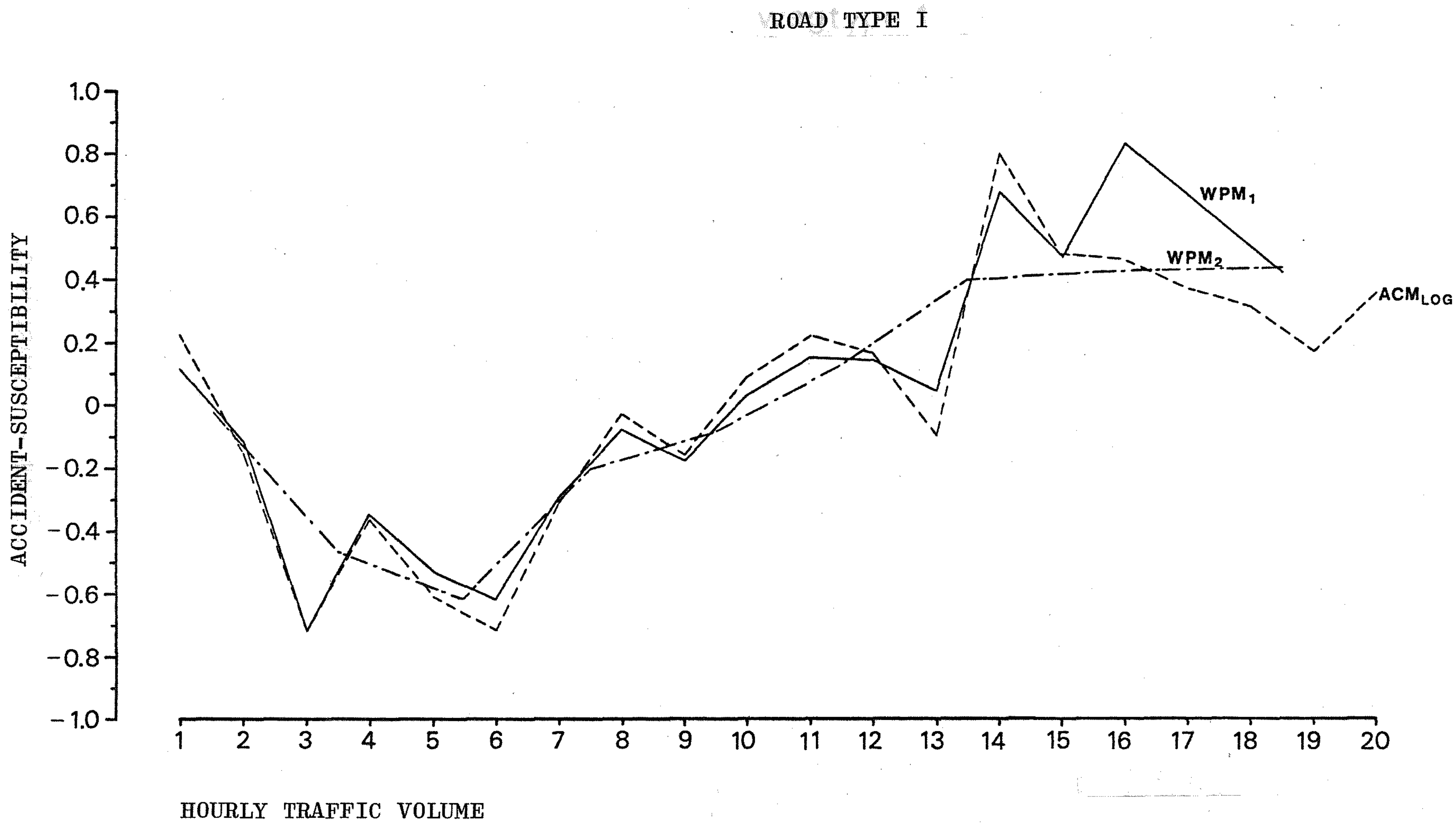

Figure 5. ACM_{log} and WPM-solution for traffic-volume classes of <u>road type I</u>.

WPM_1 corresponds to original data set, WPM_2 to totalled data set.

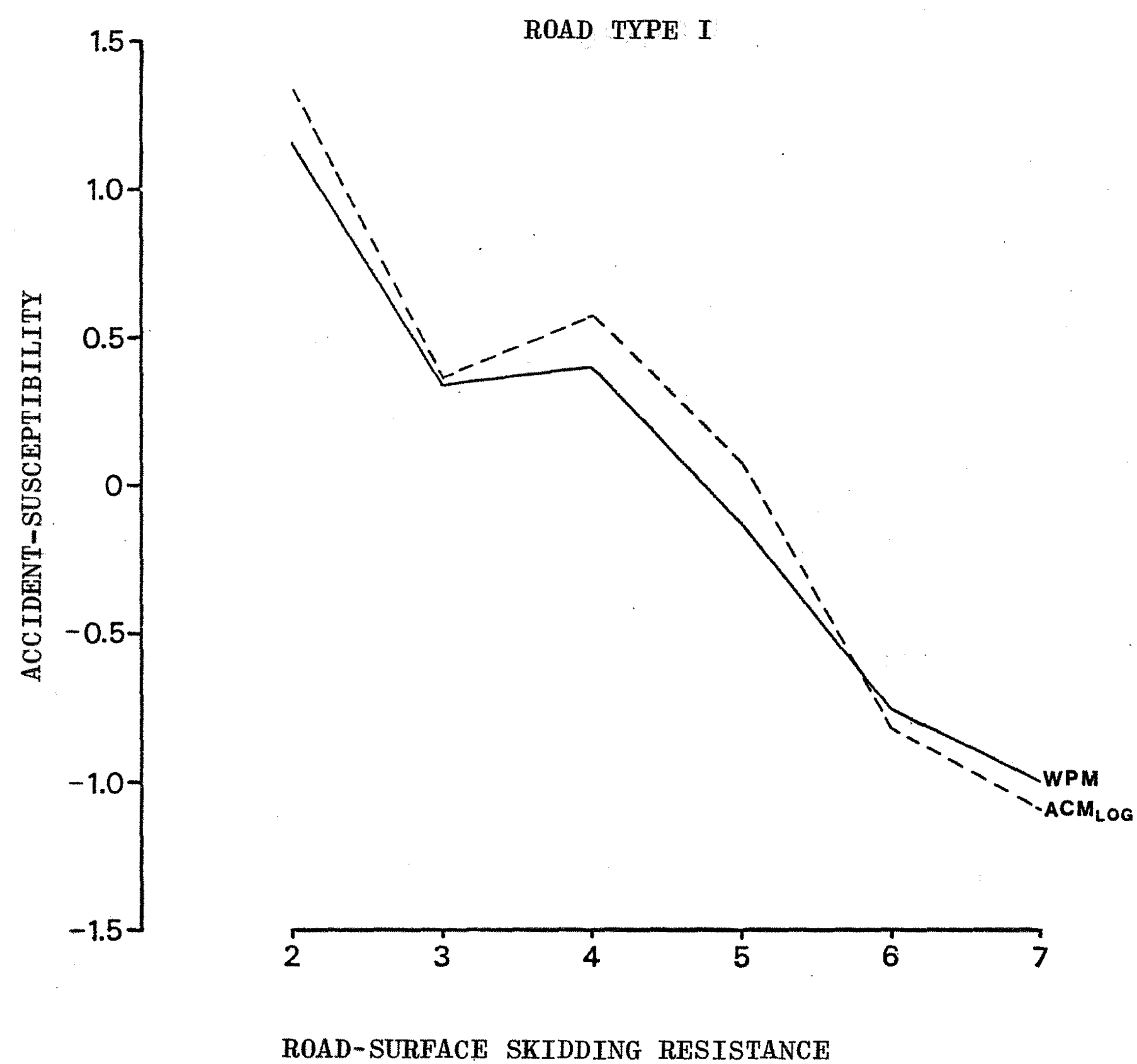

Figure 6. ACM_{log} and WPM-solution for skidding-resistance classes of road type I.

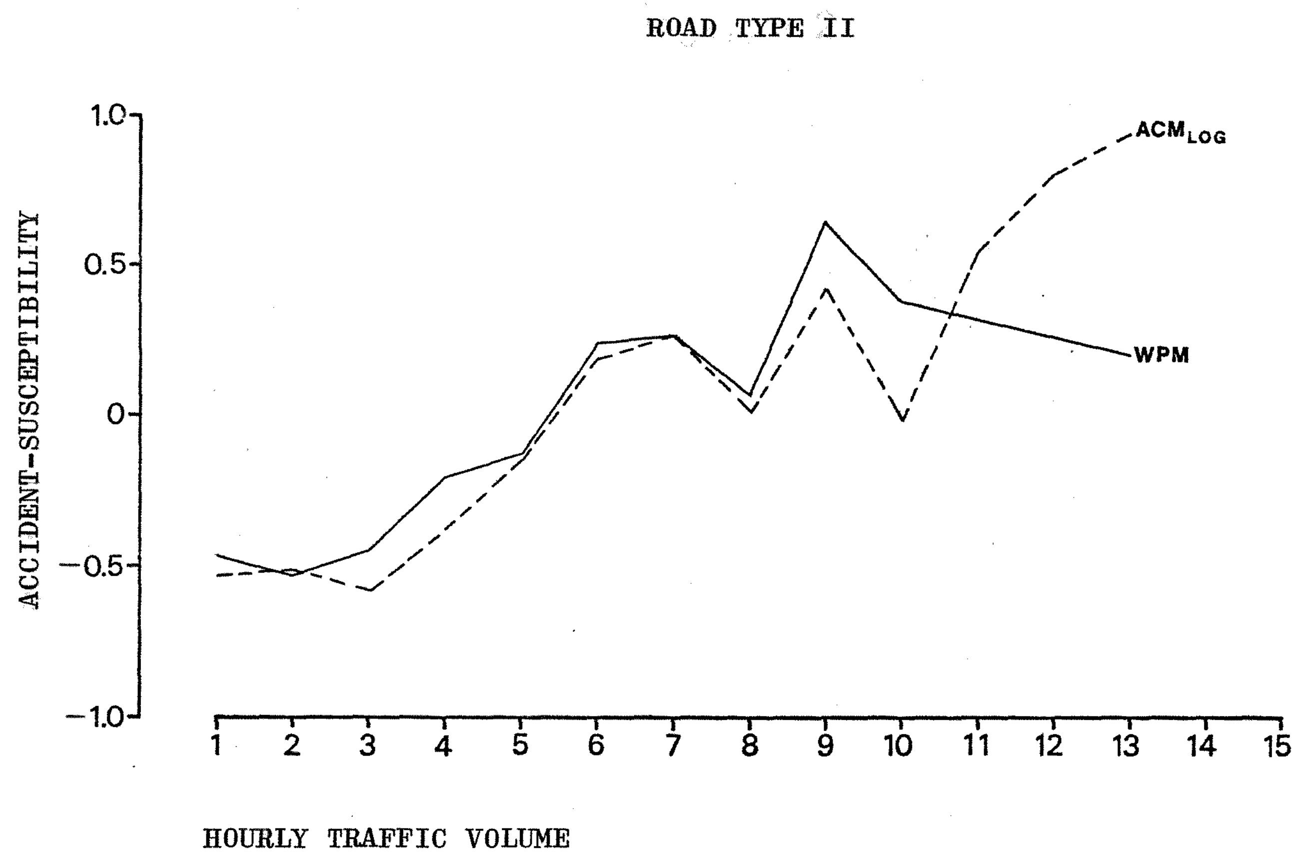

Figure 7. ACM_{log} and WPM-solution for traffic-volume classes of road type II.

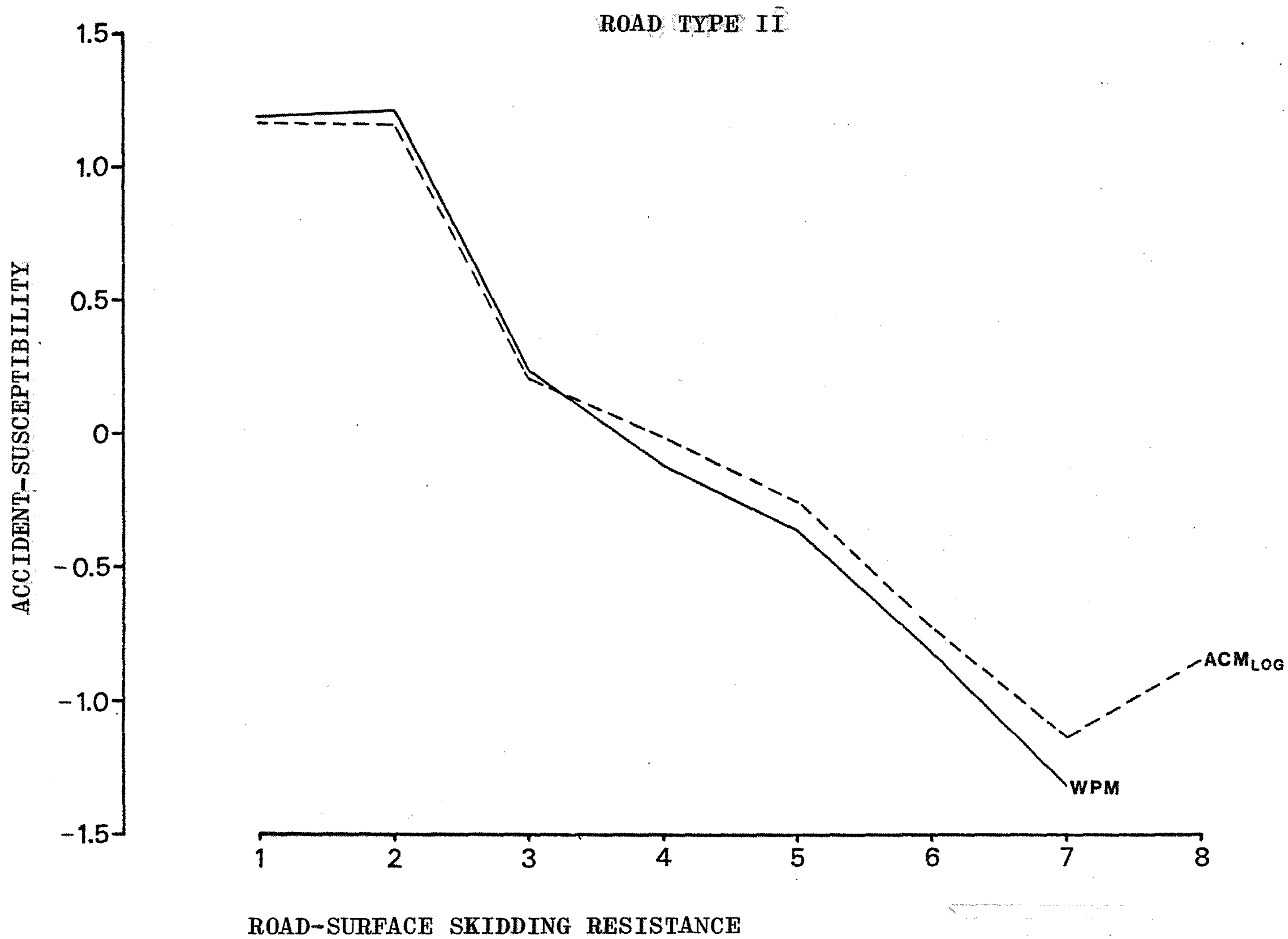

Figure 8. ACM_{log} and WPM-solution for skidding-resistance classes of road type II.